Questions on iterations.

1. Let \( f(x) = \frac{x}{1+x^2} \). The solution of the equation \( f(x) = 0 \) is \( \alpha = 0 \). The Newton iteration scheme is

\[
x_{n+1} = \frac{-2x_n^3}{1-x_n^2}, \quad n = 0, 1, 2, \ldots
\]

If \( x_n \) happens to be \( \pm 1 \) then, at the next step, the iteration blows up. What can you say about the iterations \( x_0, x_1, \ldots, x_{n-1}, \pm 1 \) that explode in finitely many steps? Is it some particular sequence? What happens if you run the Newton iterations backwards starting from \( x_n = 1 \)? Use a computer for hints and illustration.

2. Let \( f(x) = x + x^2 \sin(1/x) \) with \( f(0) = 0 \). Then \( \alpha = 0 \) is a solution of \( f(x) = 0 \). How do the Newton iterations behave? Does your answer depend on \( x_0 \)? Use a computer for hints and illustration.

3. Consider the sequence \( x_{n+1} = \cos(x_n), \ n = 0, 1, 2, \ldots \). Given \( x_0 \), how do the iterations behave? Does the answer depend on \( x_0 \)? Explain. Use a computer for hints and illustration.