Chapter 2.2 Practice Problems

EXPECTED SKILLS:

- Know how to compute the derivative of a function using the limit definition.
- Understand the geometric interpretation of a derivative (as the slope of a tangent line), and be able to use the derivative to help find the equation of a tangent line.
- Understand the physics interpretation of the derivative (as instantaneous velocity).
- Understand how the graph of a function affects the derivative.
- If given the graph of a function, be able to make a reasonable sketch of its derivative function.

PRACTICE PROBLEMS:

1. For each of the following problems, use the definition of the derivative to calculate $f'(x)$.

   (a) $f(x) = 3x$
   (b) $f(x) = 2x^2 - x$
   (c) $f(x) = 3\sqrt{x}$
   (d) $f(x) = \frac{1}{\sqrt{x}}$
   (e) $f(x) = \frac{1}{x - 1}$

2. For each of the following, sketch the graph of the given function and determine where the function is not differentiable. Explain.

   (a) $f(x) = |x + 2|$
   (b) $f(x) = \sqrt[3]{x}$
   (c) $f(x) = \begin{cases} x + 1 & \text{if } x > 1 \\ x^2 & \text{if } x \leq 1 \end{cases}$

3. Find an equation to the tangent line to the given function at the given point.

   (a) $f(x) = x^3$ at $x = 2$
   (b) $f(x) = x^2 - 1$ at $x = -1$

4. Suppose that the function $f(x)$ satisfies $f'(6) = 3$ and $f(6) = 1$. Compute an equation of the tangent line to $f(x)$ when $x = 6$. 
5. Suppose \( f(x) \) is a function such that \( f'(x) = x^2 - 4 \).

(a) For which value(s) of \( x \) will \( f(x) \) have horizontal tangent lines?

(b) For which value(s) of \( x \) will the tangent line to \( f(x) \) be parallel to the line \( y = 5x - 37 \)?

(c) For which value(s) of \( x \) will the tangent line to \( f(x) \) be perpendicular to the line \( y = 2x + \pi \)?

6. Match each of the graphs for functions (a)-(d) with the appropriate graph of its derivative (i)-(iv).

(a) \[ \text{(a)} \]

(b) \[ \text{(b)} \]

(c) \[ \text{(c)} \]

(d) \[ \text{(d)} \]

(i) \[ \text{(i)} \]

(ii) \[ \text{(ii)} \]

(iii) \[ \text{(iii)} \]

(iv) \[ \text{(iv)} \]
7. Sketch a function $y = f(x)$ with the given characteristics. (There are many possible answers.)

(a) $f'(x) < 0$ when $x < 0$; $f'(x) > 0$ when $x > 0$; and $f(0) = 0$.

(b) $f'(x) = 0$ when $x < 0$; $f'(x) < 0$ when $x > 0$; and $f(-1) = 3$; $f'(0)$ DNE.

(c) $f'(x) > 0$ when $x < -1$ and when $x > 1$; $f'(x) < 0$ when $-1 < x < 1$.

(d) $f(x)$ has a vertical tangent line when $x = 1$; $f'(x) > 0$ for $x < 1$; $f(x)$ is not differentiable when $x = -1$. 