Chapter 5.2 Practice Problems

EXPECTED SKILLS:

- Given a differentiation rule, be able to construct the associated indefinite integration rule.

- Know how to integrate polynomials, exponential functions, trigonometric functions, and inverse trigonometric functions, i.e. know Table 5.2.1.

PRACTICE PROBLEMS:

For problems 1 and 2, compute the indicated derivative and state a corresponding integration formula.

1. \( \frac{d}{dx} \left[ \frac{1}{(2x + 3)^2} \right] \)

2. \( \frac{d}{dx} [x \ln x - x] \)

For problems 3-18, evaluate given indefinite integral and check your answer by differentiation.

3. \( \int \left( \frac{1}{2}x + x^2 \right) \, dx \)

4. \( \int \left( \sqrt{x} + e \right) \, dx \)

5. \( \int \left( \frac{1}{x^3} + 3x^3 \right) \, dx \)

6. \( \int \left( 3x^{-2/3} + x^{-1/2} + 5x \right) \, dx \)

7. \( \int \left( 4x^{4/3} - 7\sqrt{x} \right) \, dx \)

8. \( \int 3 \cos x \, dx \)

9. \( \int -7 \sec^2 x \, dx \)

10. \( \int \left( -\frac{1}{x} + e^x \right) \, dx \)
11. \[ \int (1 - x^2)(x^3 + 4) \, dx \]

12. \[ \int \frac{x^2 - 3x^5}{x^3} \, dx \]

13. \[ \int \frac{-2 \sin x}{\cos^2(x)} \, dx \]

14. \[ \int \frac{1}{\sqrt{4 - 4x^2}} \, dx \]

15. \[ \int (6 \cos x + 9 \csc^2 x) \, dx \]

16. \[ \int (\sin x - 3 \sec x \tan x) \, dx \]

17. \[ \int 2^x \, dx \]

18. \[ \int \frac{x^2}{x^2 + 1} \, dx \] (HINT: Use polynomial division)

19. Consider \[ \int \cot^2 x \, dx \].
   
   (a) Using the fact that \( \sin^2 x + \cos^2 x = 1 \), derive the identity \( \cot^2 x = \csc^2 x - 1 \).
   
   (b) Use the identity that you derived in part (a) to evaluate the original integral.

For problems 20 and 21, find a function \( y = y(x) \) which satisfies the given Initial Value Problem.

20. \[
\begin{align*}
\frac{dy}{dx} &= \frac{1}{9x^2} \\
y(2) &= \frac{1}{2}
\end{align*}
\]

21. \[
\begin{align*}
\frac{dy}{dx} &= -2e^x \\
y(1) &= -5
\end{align*}
\]

22. A ball is thrown straight up in the air from an initial height of \( s_0 \) feet above the ground with an initial speed of \( v_0 \) ft/sec. Then \( s(t) \) gives the height (in feet) above the ground at time \( t \), \( v(t) = s'(t) \) gives the velocity (in ft/sec) of the ball at time \( t \), and \( a(t) = s''(t) \) gives the acceleration (in ft/sec\(^2\)) of the ball at time \( t \). Assuming that acceleration is constant, \(-g\) ft/sec\(^2\), determine \( v(t) \) and \( s(t) \).