Math 122 - Exam 2 - 2/28/2014

NAME: ______________________________

SECTION: ____________________________

Directions:

• For the free response section, you must show all work. Answers without proper justification will not receive full credit. Partial credit will be awarded for significant progress towards the correct answer. Cross off any work that you do not want graded.

• For word problems, all answers must include appropriate units in order to earn full credit.

• You have 50 minutes to complete this exam. When time is called, STOP WRITING IMMEDIATELY.

• You may not use any electronic devices including (but not limited to) calculators, cell phones, or iPods. Using such a device will be considered a violation of the university’s academic integrity policy and, at the very least, will result in a grade of 0 for the exam.

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Free Response

Reminder: For each of the following problems, you must show all of your work to earn full credit. Simplify all answers.

1. (14 Points) Evaluate the following indefinite integral:

\[ \int x^2 \ln x \, dx \]
2. (10 Points) The rectangular swimming pool shown below has a length of 20 feet and a width of 15 feet. The sides are 10 feet high. If the swimming pool is filled with water to a height of 9 feet, how much work is required to pump all of the water over the side? Recall that the weight density of water is 62.4 lb/ft$^3$.

Set up but do not evaluate an integral which can be used to solve this problem.
3. (21 Points) Let $R$ be the shaded region below.

(a) **Set up but do not evaluate** an integral (or integrals) which represent(s) the volume of the solid that results from revolving $R$ around the $x$-axis.

(b) **Set up but do not evaluate** an integral (or integrals) which represent(s) the volume of the solid that results from revolving $R$ around the $y$-axis.
(c) Consider the solid whose base is the region $R$ (from the previous page) and whose cross-sections taken perpendicular to the $x$-axis are semicircles, as depicted below.

Set up but do not evaluate an integral (or integrals) which represent(s) the volume of this solid.
Multiple Choice

Circle the letter of the best answer. Make sure your circles include just one letter. These problems will be marked as correct or incorrect; partial credit will not be awarded for problems in this section. All questions are worth 5 points.

4. Which of the following is an improper integral?

(a) \[ \int_{0}^{\pi/2} \frac{1}{2 - \sin x} \, dx \]

(b) \[ \int_{0}^{\pi/2} \frac{1}{1 - \sin x} \, dx \]

(c) \[ \int_{0}^{\pi/2} \frac{1}{1 + \sin x} \, dx \]

(d) \[ \int_{0}^{\pi/2} \frac{1}{2 - \cos x} \, dx \]

(e) \[ \int_{0}^{\pi/2} \frac{1}{1 + \cos x} \, dx \]

5. A spring has a natural length of 2 feet. A force of 30 pounds is required to stretch the spring to a length of 5 feet. Which of the following represents the work (in foot-pounds) required to stretch the spring from its natural length to a total length of 4 feet?

(a) \[ \int_{0}^{2} 6x \, dx \]

(b) \[ \int_{0}^{4} 6x \, dx \]

(c) \[ \int_{2}^{4} 6x \, dx \]

(d) \[ \int_{0}^{2} 10x \, dx \]

(e) \[ \int_{2}^{4} 10x \, dx \]
6. Suppose $f''(x)$ is continuous and let $f(0) = 2$, $f(1) = 3$, and $f'(1) = 8$. Use integration by parts to evaluate $\int_0^1 xf''(x) \, dx$.

(a) 0

(b) 1

(c) 7

(d) 9

(e) 13

7. Which of the following is the correct form of the partial fraction decomposition for $\frac{2x + 1}{(x - 1)^2(x^2 + 1)^3}$?

(a) $\frac{A}{(x - 1)^2} + \frac{B}{(x^2 + 1)^3}$

(b) $\frac{A}{x - 1} + \frac{B}{(x - 1)^2} + \frac{Cx + D}{(x^2 + 1)^3}$

(c) $\frac{A}{x - 1} + \frac{B}{(x - 1)^2} + \frac{C}{x^2 + 1} + \frac{D}{(x^2 + 1)^2} + \frac{E}{(x^2 + 1)^3}$

(d) $\frac{A}{x - 1} + \frac{B}{(x - 1)^2} + \frac{Cx + D}{x^2 + 1} + \frac{Ex + F}{(x^2 + 1)^2} + \frac{Gx + H}{(x^2 + 1)^3}$

(e) $\frac{A}{x - 1} + \frac{Bx}{x - 1} + \frac{C}{x^2 + 1} + \frac{Dx + E}{x^2 + 1} + \frac{Fx^2 + Gx + I}{x^2 + 1}$
8. Consider the region $R$, shown below, which is enclosed by $y = x^2$, $x = 0$, and $y = 1$. Which of the following represents the volume of the solid that results from revolving $R$ around the line $y = 1$?

(a) $\pi \int_0^1 (\sqrt{y} - 1)^2 \, dy$

(b) $\pi \int_0^1 (y - 1) \, dy$

(c) $\pi \int_0^1 (x^4 - 1) \, dx$

(d) $\pi \int_0^1 (1 - x^4) \, dx$

(e) $\pi \int_0^1 (1 - x^2)^2 \, dx$
9. The value of \( \int_0^\infty e^{-x} \, dx \) is:

(a) \( +\infty \)

(b) Finite and positive

(c) 0

(d) Finite and negative

(e) \( -\infty \)

10. Which of the following is the length of the curve \( y = \frac{2}{3} (x - 1)^{3/2} \) on the interval \( 1 \leq x \leq 4 \)?

(a) \( \frac{\pi}{4} \)

(b) \( 9\pi \)

(c) \( \frac{1}{4} \)

(d) \( \frac{14}{3} \)

(e) \( \frac{12\sqrt{3}}{5} \)
11. Evaluate the improper integral $\int_{0}^{1} \frac{1}{\sqrt{x}} \, dx$

(a) $-\infty$

(b) $-1$

(c) $0$

(d) $2$

(e) $+\infty$

12. If we use integration by parts to evaluate $\int x^3 \sin x \, dx$, then we should pick $u$ and $dv$ to be:

(a) $u = x^3$ and $dv = dx$

(b) $u = x^3$ and $dv = \cos x \, dx$

(c) $u = x^3$ and $dv = \sin x \, dx$

(d) $u = 1$ and $dv = x^3 \sin x \, dx$

(e) $u = \cos x$ and $dv = x^3 \, dx$
13. The partial fraction decomposition of \( \frac{3x - 1}{x^2 - 1} \) has a term of the form \( \frac{A}{x + 1} \). Find the value of \( A \).

(a) \(-3\)

(b) \(-1\)

(c) \(1\)

(d) \(2\)

(e) \(3\)

14. Evaluate \( \int \frac{2x + 1}{x^2 + 1} \, dx \)

(a) \(2 \tan^{-1} x + C\)

(b) \(\ln (x^2 + 1) + C\)

(c) \(\frac{1}{2} \ln (x^2 + 1) + C\)

(d) \(\ln (x^2 + 1) + \tan^{-1} x + C\)

(e) \(\frac{1}{2} \ln (x^2 + 1) + \frac{1}{2} \tan^{-1} x + C\)