(8 points) Where does the line \( x = 1 + t, y = 3 - t, z = 2t \) intersect the cylinder \( x^2 + y^2 = 16 \)?

\[
(1+t)^2 + (3-t)^2 = 16
\]

\[
1 + 2t + t^2 + 9 - 6t + t^2 = 16
\]

\[
2t^2 - 4t - 6 = 0
\]

\[
t^2 - 2t - 3 = 0
\]

\[
(t - 3)(t + 1) = 0
\]

\[
t = 3 \quad t = -1
\]

\( t = 3 \): \( x = 1 + 3 = 4, \ y = 3 - 3 = 0, \ z = 2(3) = 6 \)

\( t = -1 \): \( x = 1 + (-1) = 0, \ y = 3 - (-1) = 4, \ z = 2(-1) = -2 \)

Two points of intersection:

\((4, 0, 6)\) and \((0, 4, -2)\)
(1 point each) For each of the following formulas, write the letter of the picture on page 2 which corresponds to it. If there is no match, write “No Match.” It is possible that some of the pictures do not correspond to any of the formulas.

\[ x + y + z = 0 \quad \text{No Match (plane)} \]

\[ x^2 + y^2 + z^2 = 9 \quad H \text{ (sphere)} \]

\[ y + x^2 - z^2 = 0 \quad \text{No Match (hyperbolic paraboloid)} \]

\[ y^2 - x^2 - z^2 = 1 \quad B \text{ (hyperboloid of two sheets opening in the y-direction)} \]

\[ x^2 + \frac{y^2}{9} + z^2 = 1 \quad E \text{ (ellipsoid “stretched” in the y-direction)} \]

\[ y^2 - x^2 - z^2 = 0 \quad A \text{ (double cone opening in the y-direction)} \]

\[ x^2 + z^2 = 9 \quad C \text{ (circular cylinder “stacked” in the y-direction)} \]

\[ x^2 - \frac{y^2}{2} + z^2 = 1 \quad G \text{ (hyperboloid of one sheet opening in the y-direction)} \]

\[ y - x^2 - z^2 = 0 \quad F \text{ (circular paraboloid opening in the y-direction)} \]

\[ y = x^2 \quad \text{No Match (parabolic cylinder “stacked” in the z-direction)} \]
(1 point each) For each of the following formulas, write the letter of the picture on page 2 which corresponds to it. If there is no match, write “No Match.” It is possible that some of the pictures do not correspond to any of the formulas.

\[ x^2 - \frac{y^2}{2} + z^2 = 1 \]  \( G \) (hyperboloid of one sheet opening in the y-direction)

\[ y^2 - x^2 - z^2 = 1 \]  \( B \) (hyperboloid of two sheets opening in the y-direction)

\[ x^2 + y^2 + z^2 = 9 \]  \( H \) (sphere)

\[ x + y + z = 0 \]  No Match (plane)

\[ y + x^2 - z^2 = 0 \]  No Match (hyperbolic paraboloid)

\[ x^2 + \frac{y^2}{9} + z^2 = 1 \]  \( E \) (ellipsoid “stretched” in the y-direction)

\[ y - x^2 - z^2 = 0 \]  \( F \) (circular paraboloid opening in the y-direction)

\[ y = x^2 \]  No Match (parabolic cylinder “stacked” in the z-direction)

\[ x^2 + z^2 = 9 \]  \( C \) (circular cylinder “stacked” in the y-direction)

\[ y^2 - x^2 - z^2 = 0 \]  \( A \) (double cone opening in the y-direction)
(10 points) Compute \( \frac{\partial z}{\partial y} \) using implicit differentiation.

Leave your answer in terms of \( x, y, \) and \( z. \)

\[
x^3z - 3x y^2 + yz = 12
\]

\[
\frac{\partial}{\partial y} \left( x^3z - 3x y^2 + yz \right) = \frac{\partial}{\partial y} (12)
\]

\[
x^3 \frac{\partial z}{\partial y} - 6xy + y \frac{\partial z}{\partial y} + z = 0
\]

\[
\frac{\partial}{\partial y} (x^3 + y) = 6xy - z
\]

\[
\frac{\partial z}{\partial y} = \frac{6xy - z}{x^3 + y}
\]
(12 points) Find the absolute maximum and minimum values of 
\[ f(x, y) = x^2 - 4xy + 5y^2 - 8y \]
on the closed triangular region \( R \) with vertices 
(0,0), (3,0), and (3,3).

\[
\begin{align*}
\frac{\partial f}{\partial x} &= 2x - 4y = 0 \quad \Rightarrow \quad x = 2y \\
\frac{\partial f}{\partial y} &= -4x + 10y - 8 = 0 \quad \Rightarrow \quad -8y + 10y - 8 = 0 \\
&\quad \Rightarrow \quad 2y - 8 = 0 \quad \Rightarrow \quad y = 4, \ x = 8
\end{align*}
\]

Critical Point: \( (4, 8) \) (not in \( R \))

Boundary:

1. From \((0,0)\) to \((3,0)\): \( y = 0, \ 0 \leq x \leq 3 \)

\[
f(x, y) = f(x, 0) = x^2 = w(x)
\]

\[
w'(x) = 2x = 0 \quad \Rightarrow \quad x \neq 0 \quad (\text{Not in } (0,3))
\]

2. From \((3,0)\) to \((3,3)\): \( x = 3, \ 0 \leq y \leq 3 \)

\[
f(x, y) = f(3, y) = 9 - 12y + 5y^2 - 8y = 5y^2 - 20y + 9 = v(y)
\]

\[
v'(y) = 10y - 20 = 0 \quad \Rightarrow \quad y = 2 \quad \text{So check } (3,2).
\]

3. From \((0,0)\) to \((3,3)\): \( y = x, \ 0 \leq x \leq 3 \)

\[
f(x, y) = f(x, x) = x^2 - 4x^2 + 5x^2 - 8x = 2x^2 - 8x = w(x)
\]

\[
w'(x) = 4x - 8 = 0 \quad \Rightarrow \quad x = 2 \quad \text{So check } (2,2)
\]

Also check vertices 
\[
<table>
<thead>
<tr>
<th>(x, y)</th>
<th>(0, 0)</th>
<th>(3, 0)</th>
<th>(3, 3)</th>
<th>(3, 2)</th>
<th>(2, 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x, y) )</td>
<td>0</td>
<td>9</td>
<td>-6</td>
<td>-11</td>
<td>-8</td>
</tr>
</tbody>
</table>
\]

Absolute Max: 9
Absolute Min: -11
(12 points) Without eliminating the parameters, find the surface area of the portion of the cone \( r = u \mathbf{i} + u \cos v \mathbf{j} + u \sin v \mathbf{k} \) for which \( 0 \leq u \leq 2 \) and \( 0 \leq v \leq 2\pi \).

\[ \mathbf{X} = u \mathbf{i}, \quad \mathbf{Y} = u \cos v \mathbf{j}, \quad \mathbf{Z} = u \sin v \mathbf{k} \]

\[ \mathbf{r}_u = \left\langle \frac{\partial x}{\partial u}, \frac{\partial y}{\partial u}, \frac{\partial z}{\partial u} \right\rangle = \left\langle 1, \cos v, \sin v \right\rangle \]

\[ \mathbf{r}_v = \left\langle \frac{\partial x}{\partial v}, \frac{\partial y}{\partial v}, \frac{\partial z}{\partial v} \right\rangle = \left\langle 0, -u \sin v, u \cos v \right\rangle \]

\[ \mathbf{r}_u \times \mathbf{r}_v = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & \cos v & \sin v \\ 0 & -u \sin v & u \cos v \end{vmatrix} = \mathbf{i} \left( u \cos^2 v + u \sin^2 v \right) - \mathbf{j} \left( u \cos v \right) + \mathbf{k} \left( -u \sin v \right) = \left\langle u, -u \cos v, -u \sin v \right\rangle \]

\[ \| \mathbf{r}_u \times \mathbf{r}_v \| = \sqrt{u^2 + u^2 \cos^2 v + u^2 \sin^2 v} = \sqrt{2u^2} = \sqrt{2} u \]

\[ S = \iint_{D} \sqrt{2} u \ du \ dv \]

\[ = \int_{0}^{2\pi} \left[ \int_{0}^{2} \sqrt{2} u^2 \right] \ dv = 2\sqrt{2} \int_{0}^{2\pi} \ dv = 4\pi \sqrt{2} \]
(5 points each) Consider the double integral \[ \iint_R (2x - y^2) \, dA \] where \( R \) is the region bounded by \( y = 1 - x, y = x + 1, \) and \( y = 3. \)

a. Set this up as an iterated integral (or integrals), where \( dA = dy \, dx \). DO NOT EVALUATE THE INTEGRAL(S).

b. Set this up as an iterated integral (or integrals), where \( dA = dx \, dy \). DO NOT EVALUATE THE INTEGRAL(S).

You must sketch the region \( R \) to receive full credit.

\[ \begin{align*}
& \int_{-2}^{0} \int_{1-x}^{3} (2x - y^2) \, dy \, dx + \int_{0}^{2} \int_{x+1}^{3} (2x - y^2) \, dy \, dx \\
& \int_{1}^{3} \int_{1-y}^{1} (2x - y^2) \, dx \, dy
\end{align*} \]
(10 points) Evaluate the iterated integral by converting to polar coordinates.

$$
\int_{-2}^{2} \int_{0}^{\sqrt{4-x^2}} \left( x^2 + y^2 \right) \, dy \, dx
$$

$$
\int_{0}^{\pi} \int_{0}^{2} r^2 \sin \theta \, dr \, d\theta = \int_{0}^{\pi} \int_{0}^{2} r^4 \, dr \, d\theta
$$

$$
= \int_{0}^{\pi} \left. \frac{1}{5} r^5 \right|_{0}^{2} \, d\theta = \frac{32}{5} \int_{0}^{\pi} d\theta = \frac{32\pi}{5}
$$
(6 points each) Consider the triple integral $\iiint_G x\,dV$ where $G$ is the solid bounded above by the plane $z = 3$ and bounded below by the cone $z = \sqrt{3x^2 + 3y^2}$.

a. Set this up as an iterated integral in rectangular coordinates.
b. Set this up as an iterated integral in cylindrical coordinates.
c. Set this up as an iterated integral in spherical coordinates.

DO NOT EVALUATE ANY OF THE INTEGRALS.
Use the transformation $u = \frac{y}{x}, v = xy$ to find $\int \int_{R} xy \, dA$ over the region $R$ enclosed by $\frac{y}{x} = 1, \frac{y}{x} = 3, xy = 1$, and $xy = 2$.

Note: The transformation can be written as $x = \sqrt[3]{u}, y = \sqrt{uv}$.

\[ \frac{y}{x} = 1 \implies u = 1 \quad \frac{y}{x} = 3 \implies u = 3 \]

\[ xy = 1 \implies v = 1 \quad xy = 2 \implies v = 2 \]

\[ x = \frac{\sqrt{v}}{\sqrt{u}} \implies \frac{\partial x}{\partial u} = -\frac{1}{2} \sqrt{v} u^{-\frac{3}{2}}, \quad \frac{\partial x}{\partial v} = \frac{1}{2 \sqrt{u v}} \]

\[ y = \sqrt{u} \sqrt{v} \implies \frac{\partial y}{\partial u} = \frac{\sqrt{v}}{2 \sqrt{u}}, \quad \frac{\partial y}{\partial v} = \frac{\sqrt{u}}{2 \sqrt{v}} \]

\[ \frac{\partial (x, y)}{\partial (u, v)} = \begin{vmatrix} -\frac{1}{2} \sqrt{v} u^{-\frac{3}{2}} & \frac{1}{2u v} \\ \frac{\sqrt{v}}{2 \sqrt{u}} & \frac{\sqrt{u}}{2 \sqrt{v}} \end{vmatrix} = -\frac{1}{4u} - \frac{1}{4u} = -\frac{1}{2u} \]

\[ \int \int_{R} xy \, dA = \frac{1}{2} \int_{1}^{3} \int_{1}^{2} \frac{v}{u} \, dv \, du = \frac{1}{2} \int_{1}^{3} \frac{1}{2} v^2 \bigg|_{1}^{2} \frac{1}{u} \, du \]

\[ = \frac{3}{4} \int_{1}^{3} \frac{1}{u} \, du = \frac{3}{4} \ln |u| \bigg|_{1}^{3} = \frac{3}{4} (\ln 3 - \ln 1) \]

\[ = \frac{3}{4} \ln 3 \]