Math 180

Homework 1 (covering 2.1-2.5 & 4.4)
10 points

Due Week 3 at the beginning of class (Tuesday 1/18/2005 for Math 180-001 and Wednesday 1/19/2005 for Math 180-701)

Please use this sheet as a coversheet for your homework.

1. Let \( A = \{ x \in \mathbb{N} \mid x \text{ is an odd number less than 8} \} \) \( B = \{3,4,5,6,7\} \) and \( C = \{x^2 \mid x = 0,1,2\} \). Find
   a) \( A \cup B \)
   b) \( B \setminus (A \cup C) \)
   c) \( C \oplus B \)
   d) \( A \times C \)

2. Using the properties of unions, intersections and complements, simplify the following:
   a) \( ((A \cup B)^c \cup A^c)^c \)
   b) \( ((A^c \cap (A^c \cup C)^c \cap D^c)^c \)

3. Let \( A \) be the set of all integers \( \mathbb{Z} \) and let \( R \) be the relation "greater than or equal to". Determine whether \( R \) is reflexive, symmetric, antisymmetric or transitive.

4. \( A = \{-1, 0, 1, 2\} \) and \( R \) is the relation on \( A \) where \( (a,b) \in R \) if and only if \( a^2 = b^2 \). This is an equivalence relation. What are the elements of \( R \) (in other words, list the order pairs that satisfy the relation)? What are the equivalence classes?

5. True or False.
   a) \( \{3, a\} \in \{1, 3, a, b\} \)
   b) \( \{3, a\} \in \{1, \{3, a\}, 7\} \)
   c) \( \emptyset \not\subset \{1\} \) where \( \emptyset \) is the empty set.
   d) \( |P(\{5,8,0,4,3,6\})| = 32 \) where \( P \) denotes the "Power Set of"
   d) \( 18 \equiv 3 \mod 4 \)
   e) \( -30 \equiv 6 \mod 9 \)
6. Determine
   a) \( 935 \mod 8 \)
   b) \( -5780 \mod 27 \)

7. What are the equivalence classes of the relation “congruence mod 8?”

8. Draw the Hasse diagram for the partial order \( \{2, 3, 5, 15, 18, 90\} \), “\(\mid\)” or “\(a\) divides \(b\)”). What are the maximal, minimal, maximum and minimum elements?

9. Extra Credit: Draw the Hasse diagram for the partial order \( \{1, 2, 3, 5, 7, 10, 21, 30, 35, 216\} \), “\(\mid\)”). Try to draw it as neatly as you can. What are the maximal, minimal, maximum and minimum elements?