1. Determine whether the subset relation, \(\subseteq\), on the sets \{A, B, C\} is reflexive, symmetric, antisymmetric or transitive. (6 points)

Reflexive: is \(A \subseteq A\)? True by definition of subset relation

Symmetric: if \(A \subseteq B\), is \(B \subseteq A\)? Not necessarily. False

Antisymmetric: If \(A \subseteq B\) AND \(B \subseteq A\), is \(A = B\)? This can only happen if \(A\) and \(B\) are the same set. So \(A = B\). True

Transitive, \(A \subseteq B\) and \(B \subseteq C\), is \(A \subseteq C\)? True. If \(A\) is inside \(B\) and \(B\) is inside \(C\) then \(A\) must be inside \(C\).

2. Determine the following (10 points)

a) \(|P\{1,\#,7,2,\%,a,c\}|\). There are 7 elements. The formula for the cardinality of the power set is \(2^{|A|}\). \(2^7 = 128\)

b) \(|P\{7,1,\{2, a\}, c\}|\). There are 4 elements. \(2^4 = 16\)

c) \(|P\{\emptyset\}|\). There are 0 elements in the empty set. \(2^0 = 1\)

d) \(2897 \mod 10\). \(\left\lfloor \frac{2897}{10} \right\rfloor = \left\lfloor 289.7 \right\rfloor = 289\)
so \(289 = 289*10 + r\); \(r = 2897 - 289*10 = 7\)

e) \(-4738 \mod 52\). \(\left\lfloor \frac{-4738}{52} \right\rfloor = \left\lfloor -91.11 \right\rfloor = -92\)
so \(-4738 = -92*52 + r\); \(r = -4738 + 92*52 = 46\)
3. True or False. Given a brief reason why in each case. (10 points)
a) \(6 \equiv 10 \mod 8\); if this is true then 6 and 10 have the same remainder when divided by 8. or 6 – 10 is divisible by 8. But 6 – 10 = - 4. - 4 is not divisible by 8. FALSE

b) \(15 \equiv 20 \mod 9\). If this is true then 15 – 20 is divisible by 9 (or 15 and 20 have the same remainder when divided by 9). But 15 – 20 = - 5. – 5 is not divisible by 9. FALSE

c) 70 \equiv 79 \mod 3. If this is true then 70 – 79 is divisible by 3. 70 - 79 = - 9. -9 is divisible by 3. TRUE

d) \([-90.72] = -90\). FALSE. It should be -91.

e) \([-333.33] = -332\) FALSE. It should be -334

4. If A = \(\{x \in \mathbb{N} \mid x < 7\}\), B = \(\{x \in \mathbb{Z} \mid |x - 5| < 3\}\), and C = \(\{2,3\}\), find \((A \oplus B) \setminus C\).

A = \{1, 2, 3, 4, 5, 6\}  B = \{3, 4, 5, 6, 7\}  C = \{2,3\}

To find the elements in B: \(|x - 5| < 3\) is the same as \(-3 < x - 5 < 3\) which becomes

\(-3 + 5 < x < 3 + 5\) or \(2 < x < 8\)

\(A \oplus B = (A \cup B) \setminus (A \cap B) = \{1, 2, 3, 4, 5, 6, 7\} \setminus \{3, 4, 5, 6\} = \{1, 2, 7\}\)

\((A \oplus B) \setminus C = \{1, 2, 7\} \setminus \{2,3\} = \{1,7\}\)

5. Given the partial order (\(\{2, 3, 4, 5, 7, 14, 21\}\), ‘|’ or “a divides b”)
a) Draw the Hasse diagram (6 points)

2 divides 4 and 14
3 divides 3, 21
4 divides 4
5 divides 5
7 divides 14, 21
14 divides 14
21 divides 21
b) What are the maximal, minimal, maximum and minimum elements (if any)? (4 points)

maximal: 4, 14, 21, 5
minimal: 2, 3, 5, 7
maximum: None
minimum: None

6. Given the partial order \(\{\emptyset, \{a\}, \{b\}, \{a,b,c\}, \{b,c\}, \{c,d\}, \{a,b,d\}\}, \subseteq\)

a) Draw the Hasse diagram (6 points)

\[
\begin{align*}
\{a, b, d\} & \rightarrow \{a, b, c\} \\
\{b, c\} & \rightarrow \{a\} \\
\{b\} & \rightarrow \{b\} \\
\{c, d\} & \rightarrow \{c, d\} \\
\emptyset &
\end{align*}
\]

Empty set is a subset of every set
\{a\} is a subset of \{a\}, \{a, b, c\} and \{a, b, d\}
\{b\} is subset of \{b\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}
\{a, b, c\} is subset of \{a, b, c\}
\{b, c\} is subset of \{b, c\}, \{a, b, c\}
\{c, d\} is subset of \{c, d\}
\{a, b, d\} is subset of \{a, b, d\}

b) What are the maximal, minimal, maximum and minimum elements (if any)? (4 points)

Maximal: \{a, b, d\}, \{a, b, c\}, \{c, d\}
Minimal: Empty set
Maximum: None
Minimum: Empty set

7. Suppose we partitioned the integers, \(\mathbb{Z}\), into 4 sets:  
\(A_1 = \{4k \mid k \text{ is an integer}\}\),  
\(A_2 = \{4k + 1 \mid k \text{ is an integer}\}\),  
\(A_3 = \{4k + 2 \mid k \text{ is an integer}\}\),  
\(A_4 = \{4k + 3 \mid k \text{ is an integer}\}\).

This is an equivalence relation. What is this particular equivalence relation called? What are the equivalence classes? (5 points)
This is congruence mod 4.
The equivalence classes are just the partitions $A_1, A_2, A_3, A_4$

8. Shade in the region in the Venn diagram corresponding to $((A \cap B) \cup C)^c$ (5 points)

It is easier to first shade in the region before the complement; $((A \cap B) \cup C)$

The answer is everything that is not shaded above which includes the part outside of the 3 sets.

9. Write an expression for the shaded region (6 points)

One possible solution: $((B \cup C) \setminus (B \cap C)) \cup (A \cap B \cap C)$
10. If $A \subseteq B$, what is
   (4 points)
   a) $A \cup B = B$. since $A$ is already inside $B$, the union is just $B$
   b) $A \cap B = A$. since $A$ is contained in $B$, the only overlapping piece is $A$.
   c) $A \setminus B = \text{Empty set}$. If you take $B$ away from $A$, then you’ve taken $A$ from $A$.
   d) $A \oplus B = (A \cup B) \setminus (A \cap B) = B \setminus A$. This is based on (a) and (b)

Essentially the picture for this problem looks like this. You can verify (a)-(c) by shading in what is being asked.

11. Let $P$ indicate “the power set of”. Find
   a. $P\{4, a, b\}$ (5 points)
      \[
      \{ \emptyset, \{4\}, \{a\}, \{b\}, \{4,a\}, \{4,b\}, \{a,b\}, \{4,a,b\} \}
   \]
   b. $P(P\{1\})$ (5 points)

   $P\{1\} = \{ \emptyset, \{1\} \}$. There is one element in $\{1\}$. Its power set contains 2 elements:
   the empty set and the set itself.

   $P(P\{1\}) = P(\{ \emptyset, \{1\} \}) = \{ \emptyset, \emptyset, \emptyset, \{1\}, \emptyset, \{1\} \}$.

   There are 2 elements in $\{ \emptyset, \{1\} \}$. So there should be 4 elements in the power set: the empty set, the sets containing each element individually, and the entire set.

   If you are confused, pretend that $\emptyset$ is “a” and $\{1\}$ is “b”
   Then the power set of $\{a,b\}$ is $\{ \emptyset, \{a\}, \{b\}, \{a,b\} \}$, which matches what we got above.