SKETCHING THE GRAPH OF \( y = f(x) \):

1. FIND ALL POINTS AT WHICH THE BEHAVIOR OF THE GRAPH COULD CHANGE.
   
   (a) POINTS NOT IN THE DOMAIN OF \( f \) : \( f(x) \) UNDEFINED
   
   (b) POSSIBLE EXTREME POINTS : \( f'(x) \) ZERO OR UNDEFINED
   
   (c) POSSIBLE INFLECTION POINTS : \( f''(x) \) ZERO OR UNDEFINED

2. THE X-COORDINATES OF THE POINTS FROM #1 DIVIDE THE X-AXIS UP INTO INTERVALS ON WHICH THE BEHAVIOR CANNOT CHANGE, I.E., ON WHICH THE GRAPH IS ONE OF THE FOLLOWING TYPES:
   
   (a) \( f' > 0 \) AND \( f'' > 0 \) :  

   (b) \( f' > 0 \) AND \( f'' < 0 \) :  

   (c) \( f' < 0 \) AND \( f'' > 0 \) :  

   (d) \( f' < 0 \) AND \( f'' < 0 \) :  

Determine the sign of $f'$ and $f''$ on each interval by checking one point in each.

Piece these curves together, being careful to indicate any horizontal ($f' = 0$) or vertical ($f'$ undefined) tangents.

3. When necessary (and feasible) add intercepts ($x = 0$ and $y = 0$) and horizontal asymptotes ($\lim_{x \to \pm \infty} f(x)$)

**Examples:**

1. $y = f(x) = x^4 - 2x^3$ (defined everywhere)
   
   $= x^3(x-2)$ (x-intercepts at $x = 0, x = 2$)

   $f'(x) = 4x^3 - 6x^2$ (defined everywhere)
   
   $= 2x^2(2x-3)$

   $f'(x) = 0$ at $x = 0$ and $x = \frac{3}{2}$

   $y$-coordinates: $f(0) = 0$
   
   $f\left(\frac{3}{2}\right) = -\frac{27}{16}$

   $(0, 0)$ $(\frac{3}{2}, -\frac{27}{16})$

   $f''(x) = 12x^2 - 12x$ (defined everywhere)
   
   $= 12x(x-1)$

   $f''(x) = 0$ at $x = 0$ and $x = 1$

   $y$-coordinates: $f(0) = 0$

   $f(1) = -1$

   $(0, 0)$ $(1, -1)$
Now piece these together, noting that there are no horizontal asymptotes \( \lim_{x \to \pm \infty} (x^4 - 2x^3) = \infty \), and that the graph passes through \((0,0)\) and \(\left(\frac{3}{2}, -\frac{27}{16}\right)\) horizontally.