Math 123 Fall ’09 Exam 1

Wednesday, October 14, 2009

Name: _____________________________

Section: _____________________________

Instructions:

You must SHOW ALL WORK and USE CORRECT NOTATION to receive credit. Your work must be organized, legible, and unambiguous. You must simplify all of your work unless you are explicitly instructed not to.

All solutions to differential equations should be written as explicit solutions, if possible.

Do not write anything on this cover page below the following line.

1._______ 2._______ 3._______ 4._______ 5._______
6._______ 7._______ 8._______ 9._______

Total_________
1. (12 points) Solve the differential equation.

\[
\frac{dy}{dx} - y^2 \sin x = 0
\]
2. (12 points) Solve the differential equation.

\[ y' + \frac{1}{x} y = \cos x, \quad x > 0 \]
3. (12 points) Solve the differential equation.

\[ y'' - y' + 3y = 0 \]
4. (12 points) Solve the initial-value problem.

\[
\begin{align*}
\frac{dy}{dx} &= 3x^2 y^2 + 3x^2 \\
y(0) &= 1
\end{align*}
\]
5. (12 points)
Find a curve in the xy-plane that has a y-intercept of 1, a horizontal tangent line at the y-intercept, and satisfies \( y'' - y' = 6y = 0 \)
6. (12 points) A tank initially contains 20 ounces of salt dissolved in 300 gallons of water. Brine containing 2 ounces of salt per gallon enters the tank at a rate of 10 gallons per minute and the mixed solution is drained from the tank at the same rate. How much salt is in the tank at an arbitrary time $t$?
7. (12 points)

Suppose that a quantity \( y = y(t) \) increases at a rate that is proportional to the square root of the amount present, and suppose that at time \( t = 0 \) the initial amount present is \( y_0 \).

- Set up an initial-value problem for this model.
- Solve the initial-value problem and find a formula for \( y(t) \). (You may assume that \( y(t) \) is always positive.)
8. (12 points)

Suppose that 20% of a radioactive substance decays in 4 weeks. How long will it take for 90% of the substance to decay? You may leave $\ln$ in your answer.
Note: You do not have to solve the initial-value problem for the exponential decay model. You can simply begin with the appropriate equation for $y(t)$. 
9. (4 points)

Prove that the functions $y_1 = xe^{mx}$ and $y_2 = e^{mx}$ are linearly independent.