Math 123 Fall ’09 Exam 2

Wednesday, October 28, 2009

Name: _____________________________
Section: _____________________________

Instructions:
You must SHOW ALL WORK and USE CORRECT NOTATION to receive credit. Your work must be organized, legible, and unambiguous. You must simplify all of your work unless you are explicitly instructed not to.

Do not write anything on this cover page below the following line.

1.______  2.______  3.______  4.______  
5.______  6.______  7.______  8.______  

Total_______
1. (8 points each) Determine if the following sequences converge or diverge. If a sequence converges, find its limit.

a. \[ \left\{ \frac{\ln 5n}{\ln 3n} \right\}_{n=1}^{n=\infty} \]

b. \[ \left\{ \frac{\sin^2 n}{2^n} \right\}_{n=1}^{n=\infty} \] (Hint: Squeeze Theorem)
2. Consider the following sequence:

\[ \sqrt{3} - \sqrt{4}, \sqrt{4} - \sqrt{5}, \sqrt{5} - \sqrt{6}, \ldots \]

a. (2 points) Find a formula for the general term of the sequence, starting with \( n = 1 \).

b. (6 points) Determine if the sequence converges or diverges. If it converges, find its limit.
3. Consider the sequence:

\[ a_1 = \sqrt{2}, a_2 = \sqrt{2 + \sqrt{2}}, a_3 = \sqrt{2 + \sqrt{2 + \sqrt{2}}}, \ldots \]

a. (2 points) Describe the sequence with a recursive formula.

b. (6 points) Find the limit of the sequence. (You may assume that the sequence does in fact converge to a limit.)
4. (8 points each)

   a. Use a test for monotonicity to determine if the sequence is strictly increasing or strictly decreasing.

\[
\left\{ \frac{8^n}{(3n)!} \right\}_{n=1}^{\infty}
\]

   b. Use a test for monotonicity to show that the sequence is eventually strictly increasing or eventually strictly decreasing.

\[
\left\{ n^3 - 4n^2 \right\}_{n=1}^{\infty}
\]
5. (8 points each) Determine if the following series converge or diverge. If a series converges, find its sum.

a. \[\sum_{k=1}^{\infty} \left[ \arctan(k) - \arctan(k+1) \right] \]

b. \[\sum_{k=1}^{\infty} 3^{3-k} 2^{k+2} \]
6. (10 points) Determine if the following series converges or diverges. If it converges, find its sum.

\[ \sum_{k=1}^{\infty} \frac{1}{k^2 + 2k} \]
7. (8 points each) Use a convergence test and determine whether or not the series converges. **Specify which test you are using.**

a. \[ \sum_{k=7}^{\infty} 5k^{-\frac{\pi}{10}} \]

b. \[ \sum_{k=2}^{\infty} \frac{k^2 + 6}{k - 4k^2} \]
8. (10 points) Use a convergence test and determine whether or not the series converges. Specify which test you are using.

\[ \sum_{k=1}^{\infty} ke^{-k^2} \]