2) (a) Yes (b) No (c) We did not discuss complete binary trees in class so you are not responsible for knowing this (d) b (e) c has no children (f) 2 (g) 3

37) **Proof by induction.**

**Base Case:** $n = 2$ $\Rightarrow$ tree looks like: 

Both nodes have degree 1. $\therefore$ $P(2)$ is true

**Inductive Step:** Assume $P(k)$: a tree with $k$ nodes, $k \geq 2$, has at least two nodes of degree 1.

Must show $P(k+1)$: a tree with $k+1$ nodes has at least two nodes of degree 1.

Given a tree with $k+1$ nodes, remove leaf $x$ (and its arc). The resulting tree has $k$ nodes and thus by the inductive hypothesis has at least two nodes of degree one. Putting node $x$ back to obtain the original tree, we note that $x$ has degree one (because it is a leaf), but the parent of $x$ may go from degree one to degree two. So the number of nodes with degree one either increases by one or stays the same. Either way, our original tree (with $k+1$ nodes) has at least two nodes of degree one.
40) Assume we have a full binary tree with \( x \) internal nodes.

a) All internal nodes will have two children, so the total number of "children nodes" (i.e., nodes that are children of some other node) is \( 2x \). The only "non-child" node is the root. So there are \( 2x+1 \) total nodes.

b) From part (a) there are \( 2x+1 \) total nodes, \( x \) of which are internal. So there are \( (2x+1)-x = x+1 \) leaves.

c) Assume we have a full binary tree with \( n \) total nodes. Let \( x \) be the number of internal nodes. From part (a), we know \( n = 2x+1 \) \( \Rightarrow x = \frac{(n-1)}{2} \). From part (b) we know the number of leaves is \( x+1 = \frac{(n-1)}{2} + 1 = \frac{(n+1)}{2} \).

41) Assume we have a binary tree where \( x \) is the number of nodes with two children, let \( y \) be the number of nodes with one child. So the total number of "children nodes" is \( 2x+y \). The only non-child is the root. So there are \( 2x+y+1 \) total nodes, \( x+y \) of which are internal. So there are \( (2x+y+1)-(x+y) = x+1 \) leaves, which is one more than the number of nodes with two children.