A pair of fair dice is rolled.

6) Let $S$ be sample space.
   
   $|S| = 6 \cdot 6 = 36$

10) Let $E$ be event: getting 1 on at least one die

   $|E| = \frac{1 \cdot 6 + 6 \cdot 1 - 1 \cdot 1}{1 \text{ on} \ 1 \text{ on} \ 1 \text{ on}} = 11$

   $1 \text{st die} \ 2 \text{nd die} \ \text{both dice}$

   OR $|E| = \frac{1 \cdot 5 + 5 \cdot 1 + 1 \cdot 1}{1 \text{ on} \ 1 \text{ on} \ 1 \text{ on}} = 11$

   $1 \text{st die} \ 2 \text{nd die} \ \text{both dice}$

   only

   OR $|E| = \frac{36 - 5 \cdot 5}{\text{all} \ \text{no 1's}} = 11$

   possibilities

   So $P(E) = \frac{|E|}{|S|} = \frac{11}{36}$
2-card hand

13) Let $S$ be sample space
$|S| = C(52, 2) = 1326$

17) Let $E$ be event: at least one card is spade

$|E| = C(13, 1)C(39, 1) + C(13, 2)C(39, 0) = 585$

exactly one spade

exactly two spades

OR $|E| = C(52, 2) - C(13, 0)C(39, 2) = 585$

all 2-card hands

no spades

So $P(E) = \frac{|E|}{|S|} = \frac{585}{1326}$

24) Powerball: Draw five numbers between 1 and 49
and then one Powerball number between 1 and 42

Let $S$ be sample space

$|S| = 49 \cdot 48 \cdot 47 \cdot 46 \cdot 45 \cdot 42$

$= 9,610,695,360$
a) Let \( E_1 \) be event: match all five numbers and Powerball number

\[
|E_1| = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 1 = 5! = 120
\]

So \( P(E_1) = \frac{|E_1|}{181} = \frac{120}{9,610,695,360} \)

b) Let \( E_2 \) be event: match none of the five numbers but match Powerball number \( 130,320,960 \)

\[
|E_2| = 44 \cdot 43 \cdot 42 \cdot 41 \cdot 40 \cdot 1 = 44,266,024
\]

So \( P(E_2) = \frac{|E_2|}{181} = \frac{130,320,960}{9,610,695,360} \)

42) In Example 65, we are given:

\[
P(A) = 0.17, \quad P(B) = 0.34, \quad P(A \cap B) = 0.08
\]

a) \( P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.08}{0.34} \approx 0.235 \)

b) \( P(A \cup B) = P(A) + P(B) - P(A \cap B) \)

\[
= 0.17 + 0.34 - 0.08 = 0.43
\]

c) \( P(A^C \cup B) = 1 - P(A \cup B) = 1 - 0.43 = 0.57 \)
49) 3 children; boys and girls equally likely offspring.

\[ S = \{ \text{BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG} \} \]
\[ |S| = 8 \]

\( E_1 = \text{at least one boy} \implies P(E_1) = \frac{7}{8} \)

\( E_2 = \text{at least one boy and at least one girl} \implies P(E_2) = \frac{6}{8} \)

So \( P(E_2|E_1) = \frac{P(E_2 \cap E_1)}{P(E_1)} = \frac{\frac{6}{8}}{\frac{7}{8}} = \frac{6}{7} \)

53) 43 red balls, 27 green balls, 8 blue balls (78 total)

Guessing correct: Red = $3, Green = $6, Blue = $10

(a) Expected value: \( (\frac{43}{78})(3) + (\frac{27}{78})(6) + (\frac{8}{78})(10) = \$4.76 \)

(b) Price of game is $5,

so expected profit for casino is \( $5 - $4.76 = $0.24 \)

per game. After 100 games, the casino expects to make \( (100)(0.24) = $24 \)