11) The equivalence relations are (10b), (10e), (10f), (10h).

The equivalence classes are:

(10b): \[ \{ \ldots, -6, -3, 0, 3, 6, \ldots \} = \{ x \mid x = 3k \text{ for some } k \in \mathbb{Z} \} \]
\[ \{ \ldots, -5, -2, 1, 4, 7, \ldots \} = \{ x \mid x = 3k + 1 \text{ for some } k \in \mathbb{Z} \} \]
\[ \{ \ldots, -4, -1, 2, 5, 8, \ldots \} = \{ x \mid x = 3k + 2 \text{ for some } k \in \mathbb{Z} \} \]

Note that the relation in (10b) could have been written as: \( x \sim y \Leftrightarrow x \equiv y \pmod{3} \)

(10e): the sets of squares with equal length sides
(10f): the sets consisting of strings with the same number of characters
(10h): the sets consisting of sets with the same number of elements

20c)

\begin{align*}
\{a, b^3\} & \sim \{a, c^3\} \\
\{a^3\} & \sim \{c^3\} \\
\{b^3\} & \sim \{a^3\} \\
\{\emptyset\} & \sim \{\emptyset\}
\end{align*}

21c)
Least: \( \emptyset \)
Greatest: None
Minimal: \( \emptyset \)
Maximal: \( \{a, b^3, \{a, c^3\}\} \)
23) (a)  

210  
/  
/  
42  105  
/  
/  
2  21  
/  
/  
3  7  
/  
/  
2  3  7  5

Least: None  
Greatest: 210  
Minimal: 2,3,5,7  
Maximal: 210

26) Given $(S, \rho)$ and $(T, \sigma)$ are posets.

Given $(s_1, t_1) \leq (s_2, t_2) \iff s_1 \rho s_2$ and $t_1 \sigma t_2$

Prove that $\mu$ is a partial ordering on $S \times T$.

Must show that $\mu$ is reflexive, anti-symmetric, and transitive.

Reflexive: let $(s_1, t_1) \in S \times T$ (Must show $(s_i, t_i) \mu (s_i, t_i)$)

Now $s_i \rho s_i$ and $t_i \sigma t_i$ (b/c $\rho, \sigma$ are reflexive)

$\Rightarrow (s_i, t_i) \mu (s_i, t_i)$ QED

Anti-symmetric: Assume $(s_1, t_1) \mu (s_2, t_2)$ and $(s_2, t_2) \mu (s_1, t_1)$

(Must show $(s_1, t_1) = (s_2, t_2)$)

$\Rightarrow s_1 \rho s_2, t_1 \sigma t_2, s_2 \rho s_1, t_2 \sigma t_1$

$s_1 = s_2, t_1 = t_2$  
(b/c $\rho, \sigma$ are anti-symmetric)

$\Rightarrow (s_1, t_1) = (s_2, t_2)$ QED
Transitive: Assume \((s_1, t_1) \in (s_2, t_2) \) and \((s_2, t_2) \in (s_3, t_3)\)  
(Must show \((s_1, t_1) \in (s_3, t_3)\))

\[ \Rightarrow s_1p_1s_2, t_1o_1t_2, s_2p_2s_3, t_2o_2t_3 \]

\[ \Rightarrow s_1p_1s_3, t_1o_1t_3 \]

(because \(p, o\) are transitive)

\[ \Rightarrow (s_1, t_1) \in (s_3, t_3) \quad \text{QED} \]

36 b) \(\{ (a, a), (b, b), (c, c), (d, d), (e, e), (a, b), (b, a), (a, c), (c, a), (b, c), (c, b), (d, e), (e, d) \}\)

40) Given \(x py \iff x^2 - y^2\) is even

Prove that \(p\) is an equivalence relation on \(S = \mathbb{N}\).

Must show that \(p\) is reflexive, symmetric, and transitive.

Reflexive: let \(x \in S\) \((\text{Must show } x px)\)

Now \(x^2 - x^2 = 0\), which is even

So \(x px\). QED
Symmetric: Assume \( xpy \) (Must show \( ypx \))

\[
\Rightarrow x^2 - y^2 \text{ is even} \\
\Rightarrow x^2 - y^2 = 2k \text{ for some } k \in \mathbb{Z} \\
\Rightarrow y^2 - x^2 = -2k = 2(-k) \\
\Rightarrow y^2 - x^2 \text{ is even } \quad (\text{by } -k \in \mathbb{Z}) \\
\Rightarrow ypx \quad \text{QED}
\]

Transitive: Assume \( xpy \) and \( ypz \) (Must show \( xpz \))

\[
\Rightarrow x^2 - y^2 \text{ is even and } y^2 - z^2 \text{ is even} \\
\Rightarrow x^2 - y^2 = 2k \text{ and } y^2 - z^2 = 2j \text{ for some } j, k \in \mathbb{Z}
\]

Now \( x^2 - z^2 = (x^2 - y^2) + (y^2 - z^2) \)

\[
= 2k + 2j \\
= 2(k + j) \\
\Rightarrow x^2 - z^2 \text{ is even } \quad (\text{by } k + j \in \mathbb{Z}) \\
\Rightarrow xpz \quad \text{QED}
\]

43)

(a) \( P_i = 1 \).

There is only one way to partition a one-element set: there will be only one block, with the lone element in that block.
b) Find $P_3$. The partitions are:

- all elements in one block = 1
- 2 elements in one block, 1 element in another = $C(3, 2) = 3$
- each element in its own block = 1

Total = 5

OR by drawing all the partitions:

- all elements in one block

- 2 in one block, 1 in another

- each element in its own block

Total = 5

$\therefore P_3 = 5$
C) Find \( P_4 \). The partitions are:

- all in one block = 1

- 3 elements in one block, 1 in another = \( \binom{4}{3} = 4 \)

- 2 elements in one block, 2 in another = \( \binom{4}{2} = 3 \)

- 2 elements in one block, 1 in a 2nd block, 1 in a 3rd block = \( \binom{4}{1} = 6 \)

- each element in its own block = 1

Total = 15

OR by drawing all the partitions:

- all in one block: \( \begin{array}{ccc}
    a & b \\
    cd
\end{array} \)

- 3 in one block, 1 in another: \( \begin{array}{ccc}
    a & b \\
    c & d
\end{array} \)

- 2 in one block, 2 in another: \( \begin{array}{ccc}
    a & b \\
    c & d
\end{array} \)

- 2 in one block, 1 in a 2nd block, 1 in a 3rd block:

- each element in its own block: \( \begin{array}{ccc}
    a & b \\
    c & d
\end{array} \)  

\[ \therefore P_4 = 15 \]