41. Democrat is twice as likely as Republican. Republican is four times as likely as Independent.

a) \( D \) = event that Democrat elected
\( R \) = " " " Republican " "
\( I \) = " " " Independent elected " "

Probability distribution:

<table>
<thead>
<tr>
<th>( x_i )</th>
<th>( D )</th>
<th>( R )</th>
<th>( I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p(x_i) )</td>
<td>( \frac{8}{13} )</td>
<td>( \frac{4}{13} )</td>
<td>( \frac{1}{13} )</td>
</tr>
</tbody>
</table>

b) \( P(D) = \frac{8}{13} \)

c) \( P(R') = 1 - P(R) = 1 - \frac{4}{13} = \frac{9}{13} \)
49) 3 children; boys and girls equally likely offspring.

\[ S = \{ BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG \} \]

\[ |S| = 8 \]

\[ E_1 = \text{at least one boy} \Rightarrow P(E_1) = \frac{7}{8} \]

\[ E_2 = \text{at least one boy and at least one girl} \Rightarrow P(E_2) = \frac{6}{8} \]

So \[ P(E_2|E_1) = \frac{P(E_2 \cap E_1)}{P(E_1)} = \frac{\frac{6}{8}}{\frac{7}{8}} = \frac{6}{7} \]

53) 43 red balls, 27 green balls, 8 blue balls (78 total)

Guessing correct: Red = $3, Green = $6, Blue = $10

(a) Expected value: \( (\frac{43}{78})(3) + (\frac{27}{78})(6) + (\frac{8}{78})(10) \approx 4.76 \)

(b) Price of game is $5,

so expected profit for casino is \( 5 - 4.76 = 0.24 \) per game. After 100 games, the casino expects to make \( (100)(0.24) = 24 \)
Section 3.6

5) The fifth term in \((3a+2b)^7\) is:

\[ C(7,4)(3a)^3(2b)^4 = 35(3)^3a^3(2)^4b^4 \]
\[ = 15120a^3b^4 \]

14) Prove \(C(n+2, r) = C(n, r) + 2C(n, r-1) + C(n, r-2)\) for \(2 \leq r \leq n\).

Recall Pascal's formula: \(C(n, k) = C(n-1, k-1) + C(n-1, k)\)

So \(C(n+2, r) = \underbrace{C(n+1, r-1) + C(n+1, r)}_{\text{Same}}\)

\[ C(n, r-2) + C(n, r-1) \]

\[ \Rightarrow C(n, r-1) + C(n, r) \]

\[ \therefore C(n+2, r) = C(n, r-2) + 2C(n, r-1) + C(n, r) \]
16. Prove \( C(n, 0) - C(n, 1) + C(n, 2) - \ldots + (-1)^n C(n, n) = 0 \)

\[
0 = 0^n = (1 + (-1))^n
\]

\[
= C(n, 0)(1)^n(-1)^0 + C(n, 1)(1)^{n-1}(-1)^1 + C(n, 2)(1)^{n-2}(-1)^2 + \ldots + C(n, n)(1)^0(-1)^n
\]

\[
= C(n, 0) - C(n, 1) + C(n, 2) - \ldots + C(n, n) (-1)^n
\]

Note that since the power of \((-1)^n \) decreases by one for each term, the terms will alternate between positive and negative.