6) \( S = \{0, 2, 4, 6, 3\} \) \( T = \{1, 3, 5, 7, 3\} \)

b) \( \{(6,3), (2,1), (0,3), (4,5)\} \)
   - This is a function with domain \( S \) and codomain \( T \).
   - It is not a one-to-one function because \( 0, 6 \in S \) have the same image in \( T \), namely \( 3 \).
   - It is not an onto function because \( 7 \in T \) has no preimage under the function.

d) \( \{(2,1), (4,5), (6,3)\} \)
   - This is not a function with domain \( S \) and codomain \( T \) because \( 0 \in S \) is not associated with any element in \( T \).

8) \( f: \mathbb{N} \to \mathbb{N} \) where \( f \) is defined by
   \[ f(x) = \begin{cases} 
   x/2 & \text{if } x \text{ is even} \\
   x+1 & \text{if } x \text{ is odd}
   \end{cases} \]
   - \( f \) is a function.
   - \( f \) is not one-to-one because \( f(1) = f(4) = 2 \).
   - \( f \) is onto.
8) \( f : \mathbb{N} \to \mathbb{N} \) where \( f \) is defined by
\[
f(x) = \begin{cases} 
  x+1 & \text{if } x \text{ is even} \\
  x-1 & \text{if } x \text{ is odd}
\end{cases}
\]
- \( f \) is a function
- \( f \) is one-to-one \( \Rightarrow f \) is a bijection and must have an inverse

Note that \( \forall x, y \in \mathbb{N} \) if \( f(x) = y \) then \( f(y) = x \).
Ex: \( f(0) = 1 \) and \( f(1) = 0 \), \( f(2) = 3 \) and \( f(3) = 2 \), etc.

We want an inverse function \( f^{-1} : \mathbb{N} \to \mathbb{N} \) such that if \( f(x) = y \) then \( f^{-1}(y) = x \). But \( f \) itself already has this property. So \( f^{-1} = f \), i.e., \( f \) is its own inverse.

23) \(-7 \mod 3 = 2 \) because \(-7 = 3(-2) + 2\)

36) \( f = (c, a, b, d) = (a \ b \ c \ d) \leftrightarrow \text{array form} \)
- \( \text{cycle form} \)

39) \((1, 2) \circ (1, 3) \circ (1, 4) \circ (1, 5)\)
\[
\begin{align*}
1 &\rightarrow 5 \rightarrow 5 \rightarrow 5 \rightarrow 5 \\
2 &\rightarrow 2 \rightarrow 2 \rightarrow 2 \rightarrow 2 \\
3 &\rightarrow 3 \rightarrow 3 \rightarrow 1 \rightarrow 2 \\
4 &\rightarrow 4 \rightarrow 1 \rightarrow 3 \rightarrow 3 \\
5 &\rightarrow 1 \rightarrow 4 \rightarrow 4 \rightarrow 4
\end{align*}
\]
\( \text{working right to left} \)
\[= (1, 5, 4, 3, 2) \]
43) \( S = \{p, q, r\} \quad T = \{k, l, m, n\} \)

a) Total number of functions:
\[
4 \cdot 4 \cdot 4 = 4^3 = 64
\]

b) Total number of injective functions:
\[
4 \cdot 3 \cdot 2 = P(4, 3) = 24
\]