These graphs are not isomorphic. Graph (b) has a loop (arc e8), and graph (a) has no loops.

These graphs are isomorphic. Since they are both simple graphs, we only need to give one bijection mapping the nodes from one graph to nodes in the other.

One possible bijection: $f: 1 \rightarrow a$

$2 \rightarrow c$

$3 \rightarrow e$

$4 \rightarrow b$

$5 \rightarrow d$
17) Draw all the non-isomorphic, simple, graphs with three nodes.

(1)\hspace{1cm} (2)\hspace{1cm} (3)\hspace{1cm} (4)

22) Prove that the following graph is planar:

Since we can draw the graph such that the arcs only intersect at the nodes, the graph is planar.

30) Determine if the following graph is planar.

This graph doesn't look like $K_5$ in its present form, but note that nodes 1, 2, 3, 4, 5 each have degree 4, which is what all nodes in $K_5$ would have.
So we can draw:

The solid lines are a subgraph of our original graph. If we added the dashed lines we'd have $K_5$, but it would no longer be a subgraph. We can form elementary subdivisions by adding nodes 6 and 7 and arcs 1-6, 6-4, 1-7, 7-3.

So finally we have:

This graph is a subgraph of our original graph, and it is homeomorphic to $K_5$.

32) Write the adjacency matrix for:

\[
\begin{pmatrix}
1 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 \\
1 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 & 0 \\
\end{pmatrix}
\]

You may also write this in lower triangular form.

47) Adjacency list for above graph:

<table>
<thead>
<tr>
<th>Node</th>
<th>Neighbors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2, 3</td>
</tr>
<tr>
<td>2</td>
<td>1, 3, 4</td>
</tr>
<tr>
<td>3</td>
<td>2, 1, 4</td>
</tr>
<tr>
<td>4</td>
<td>2, 3</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>
41) Prove that the number of leaves in any binary tree is one more than the number of nodes with two children.

Let \( n \) = # nodes with two children. We will do a proof by induction on \( n \).

**Base Case:** \( n = 0 \). Our tree will have a "chain" structure, i.e. it will look like \( \) so there will only be one leaf. So the property holds.

**Inductive Hypothesis:** Assume the property holds for \( n = k \), i.e. a binary tree that has \( k \) nodes with two children has \( k+1 \) leaves. Now suppose we have a binary tree that has \( k+1 \) nodes with two children. We must show that this tree has \( (k+1)+1 = k+2 \) leaves.

The \( k+1 \) nodes with two children must be interior nodes, since leaves have no children. Find one of these \( k+1 \) nodes that has maximum depth, i.e. it is "furthest" from the root.
This node will have two children. Remove the entire subtree rooted at one of these two children. So either a single node is removed or a finitely long chain is removed.

Either way, the new graph will now have $k$ nodes with two children (and by the inductive hypothesis $k+1$ leaves) and it will have one less leaf than the original.
Thus the original graph has $(k+1)+1 = k+2$ leaves.