Examples of solutions of the KdV equation using evolutionary vessels

Andrey Melnikov

In this talk I am going to present some classes of solutions of the KdV equation $q'_t = q'_x q + q'''_{xxx}$ constructed from evolutionary Sturm-Liouville vessels. This will include: 1. Rational solutions, Solitons, polynomial-trigonometric, exponential-polynomial solutions arising from finite dimensional realizations of vessels, 2. Classical solutions based on the inverse scattering of the SL equation (Fadeev) $-y''_{xx} + q(x,t)y = \lambda y$, 3. Periodic solutions arising from realizations of vessels on $H = \ell^2$, 4. An approach to construction of almost-periodic solutions. These solutions are known to exist in general (Lax), but if we take the initial condition $q(x,0) = \sin(x) + \sin(\sqrt{2}x)$, no one knows whether a solution of KdV exists with this initial concrete value.

These approach is general in the sense that many more (completely integrable) PDEs can be solved in this way, and evolutionary vessels are essentially generalize the Zakarov-Sabbath scheme.