Rational Inner Functions on the Disk and on Polydisks

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Fix a rational inner function $f$ on $D$ with degree $N$. If one chooses any $N + 1$ distinct points $x_1, \ldots, x_{N+1}$ on $D$, then the Nevanlinna-Pick problem on $D$ with data $x_1, \ldots, x_{N+1}$ and $f(x_1), \ldots, f(x_{N+1})$ has a unique solution. Furthermore, essentially every Nevanlinna-Pick problem on $D$ with a unique solution arises this way. In this talk, we give some examples of Nevanlinna-Pick problems on $D^n$ with $n > 1$ demonstrating the ways in which this behavior of rational inner functions on $D$ fails to extend to $D^n$. We then introduce some definitions and theorems demonstrating the ways in which this behavior extends to 1 dimensional algebraic varieties passing through $D^2$. 