Abstract
Motivated by the burgeoning of different types of manifold-valued data in areas of science and engineering, for example in diffusion tensor imaging and collaborative motion modeling, there has been a surge of interest in approximation methods for data that takes values on manifolds. In the analysis of approximation algorithms for manifold-valued data, a powerful method called proximity analysis has been developed. A set of results in this subject says that if a nonlinear approximation scheme for manifold-valued data is constructed out of a linear approximation scheme for real-valued data, and the nonlinear scheme can be shown to satisfy a so-called proximity condition to the underlying linear scheme, then the nonlinear scheme inherits certain regularity and approximation order properties of the linear scheme.

In this talk, we show how proximity analysis can be applied to solve a conjecture by David Donoho. In this conjecture, a specific approximation scheme for manifold-valued data, called the log-exp subdivision scheme, was once believed to always inherit the regularity of the underlying linear scheme. We show that this conjecture is only true up to Hölder regularity of degree $5 - \varepsilon$. Beyond degree 5, the smoothness inheritance is shown to be obstructed by the presence of curvature. One part of the analysis involves a long and annoying calculation which shows that the conjecture is true up to degree $5 - \varepsilon$, but at degree 5 and higher, Riemann’s curvature tensor manifests itself through the proximity analysis, and the non-vanishing of which disallows us to prove the conjecture. (The need of such a long and annoying calculation seems to be justified, considered Riemann himself also had to do a long and tedious calculation to discover his celebrated curvature tensor.) Another part of the analysis officially disproves Donoho’s conjecture based on analyzing the behavior of a nonlinear dynamical system at a stable fixed point. This nonlinear dynamical system has a resonance term which makes it behave differently from the linearized system; the resonance term is caused exactly by the presence of curvature.