Quiz 3: Solutions

Problem 1. Find parametric equations for the line passing through $P_1(4,2)$ and $P_2(1,3)$.

Solution. The vector $\overrightarrow{P_1P_2} = (1 - 4, 3 - 2) = (-3, 1)$ can serve as a direction vector for the line, so that we can write the parametric equations for the line through $P_1(4,2)$ parallel to $\overrightarrow{P_1P_2} = (-3, 1)$:

$$x = 4 - 3t, \quad y = 2 + t, \quad -\infty < t < \infty.$$ 

Problem 2. Find (a) the equation of the plane through $P(1,1,1)$, $Q(2,4,3)$, and $R(-1,-2,-1)$; (b) the distance from the point $S(1,2,6)$ to the plane.

Solution. (a) The normal vector of the plane, $\vec{N}$, must be orthogonal to vectors $\vec{PQ} = (2 - 1, 4 - 1, 3 - 1) = (1, 3, 2)$ and $\vec{PR} = (-1 - 1, -2 - 1, -1 - 1) = (-2, -3, -2)$. Thus we can set $\vec{N}$ to be equal to the cross product of those vectors:

$$\vec{N} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \overrightarrow{i} & \overrightarrow{j} & \overrightarrow{k} \\ 3 & 2 & 1 \\ -3 & -2 & -2 \end{vmatrix} = -2 \overrightarrow{j} + 3 \overrightarrow{k}.$$ 

The equation of the plane through $P(1,1,1)$ with normal vector $\vec{N} = (0, -2, 3)$ is $-2(y - 1) + 3(z - 1) = 0$, or equivalently,

$$-2y + 3z - 1 = 0.$$ 

(b) The distance from $S(1,2,6)$ to this plane equals

$$\frac{|-2 \cdot 2 + 3 \cdot 6 - 1|}{\sqrt{(-2)^2 + 3^2}} = \frac{13}{\sqrt{13}} = \sqrt{13}.$$