Problem 1. Given matrices

\[ A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, \]

find (a) \( A \odot B \); (b) \( B \odot A \).
Problem 2. Use a proof by cases to show that \( \lfloor n/2 \rfloor \lceil n/2 \rceil = \lfloor n^2/4 \rfloor \) for all integer \( n \).
Problem 3. Prove that if $m$ and $n$ are positive integers and $x$ is a real number, then
\[
\left\lfloor \frac{|x| + n}{m} \right\rfloor = \left\lfloor \frac{x + n}{m} \right\rfloor.
\]
Problem 4. Use mathematical induction to prove that $n! < n^n$ whenever $n$ is a positive integer greater than 1.
**Problem 5.** Give a recursive definition of the sequence \( \{a_n\} \), \( n = 1, 2, \ldots \) if (a) \( a_n = 4n - 2 \); (b) \( a_n = 1 + (-1)^n \); (c) \( a_n = n(n + 1) \); (d) \( a_n = n^2 \).
Problem 6. Let

\[ A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \]

Show that

\[ A^n = \begin{bmatrix} f_{n+1} & f_n \\ f_n & f_{n-1} \end{bmatrix} \]

whenever \( n \) is an integer greater than 1, where \( f_n, n = 1, 2, \ldots \), are the Fibonacci numbers.

**Hint.** Use mathematical induction.
Problem 7. How many strings of four decimal digits
(a) do not contain the same digit twice?
(b) end with an even digit?
(c) have exactly three digits that are 9s?
**Problem 8.** There are 38 different time periods during which classes at a university can be scheduled. If there are 677 different classes, how many different rooms will be needed?
Problem 9. A club has 25 members.

(a) How many ways are there to choose four members of the club to serve as an executive committee?

(b) How many ways are there to choose a president, vice president, secretary, and treasurer of the club?
Problem 10. What is the probability that a positive integer not exceeding 100 selected at random is divisible by 5 or 7?
Problem 11 (extra credit). Find the solution to each of the following recurrence relations and initial conditions.

(a) $a_n = 3a_{n-1}$, $a_0 = 2$.

(b) $a_n = a_{n-1} + 2$, $a_0 = 3$.

(c) $a_n = a_{n-1} + n$, $a_0 = 1$. 