Section 13.7: Tangent Planes

Intuitively, the tangent plane to a surface at a particular point is composed of all of the tangent lines to all curves on the surface through said point.

**Trick:** $\nabla F(x,y,z)$ is normal to the level surface of $w = F(x,y,z)$ through the point $(x,y,z)$ at $(x_0,y_0,z_0)$.

Recall: All we need to define a plane is a vector normal to the plane, and a point on the plane.

If $F(x,y,z)$ has continuous first order partials and $P(x_0,y_0,z_0)$ is a point on the level surface $S: F(x,y,z) = c$, then $\nabla F(x_0,y_0,z_0)$ is normal to $S$ at $P$ (assuming $\nabla F(x_0,y_0,z_0) \neq 0$).
TANGENT PLANE TO $S$ AT $(x_0, y_0, z_0)$:

$$F_x(x_0, y_0, z_0)(x-x_0) + F_y(x_0, y_0, z_0)(y-y_0) + F_z(x_0, y_0, z_0)(z-z_0) = 0$$

**EX: FIND AN EQUATION OF THE PLANE TANGENT TO THE SURFACE $S$: $x^2y - 4z^2 = -7$ AT THE POINT $P(-3, 1, -2)$**

**LET** $F(x, y, z) = x^2y - 4z^2$

$$\nabla F(x, y, z) = \langle 2xy, x^2, -8z \rangle$$

$$\nabla F(-3, 1, -2) = \langle 2(-3)(1), (-3)^2, -8(-2) \rangle$$

$$= \langle -6, 9, 16 \rangle$$

**PLANE: $-6(x+3) + 9(y-1) + 16(z+2) = 0$**