ACHIEVING WIDE FIELD OF VIEW USING DOUBLE-MIRROR CATADIOPTIC SENSORS

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ABSTRACT

For many applications such as surveillance, medical imaging, photography, and robot navigation, it is required that camera have a wide field of view. Traditional approaches to solve this problem include using a rotating camera, stitching images, complex lenses or multiple cameras. We are proposing a catadioptric sensor with a camera mirror pair for enhancing the field of view. Devices consisting of a reflective surface (catadioptric) and a camera (dioptrics) are called catadioptric sensors. Our single viewpoint double-mirror system forms a conic mirror coupled with a proper secondary mirror as a solution to a nonlinear first order differential equation. The ordinary differential equations are obtained as geometric solutions to the problem. Our system is designed to image a plane at infinity, without distortion, requiring no digital unwarping. In this work, we are analyzing the family of conics and corresponding secondary mirrors to obtain a correct image and a large field of view.

Keywords: Catadioptric sensor, folded system, orthographic projection, perspective projection, central mirrors.

PROBLEM STATEMENT

We want design a double-mirror catadioptric sensor such that the sensor have an ultra-wide field of view. Since the catoptric component of the sensor is rotationally symmetric, we can depict the problem in 2D. The primary mirror has been chosen to be a concave up cone of different slopes and projection is orthographic and perspective. Secondary mirror was designed according to the projection realized by the primary mirror.

APPLICATIONS OF CATADIOPTIC SENSORS

Applications of catadioptric sensors include surveillance, mobile autonomous robotics, as seen in the pictures, a robot has to see 360 degrees in order to navigate. There is a region for robot soccer. Also medical imaging is an area in which catadioptric sensors are widely used.

CONICAL PRIMARY MIRRORS

Our designing scheme is as follows: we start with a choice of primary mirror and a projection. Visualizing the problem in 2 dimensions, we derive the ordinary differential equations for the secondary mirror by geometric methods. Solving the differential equations either numerically or analytically, gives us the graph of the cross section of the secondary mirror.

Assumptions: The viewpoint which is a plane y=k is at an infinite distance, the optical axis is the y axis, and the camera is placed on y axis. Special case for perspective camera, the center of projection is at the point (0,1), d is the scaling constant of the viewpoint, which can be specified.

Let us define light rays are going to hit the primary mirror and then reflect off to hit the secondary mirror and bounce off towards the viewpoint.

RIGHT ANGLE MIRROR UNDER ORTHOGRAPHIC PROJECTION

Let us define vectors u and v;

\[ u = -4, x_1 > 0 \text{ and } v = kx_1, k > 0, v < x < kx_1 \text{.} \]

Normalizing u and v and taking the limit of vector \( v \rightarrow \infty \), since the view plane \( y=k \) is infinitely far, gives us: \( u = 1/2x_1^{1/2}, -1/2x_1^{-1/2} \text{ and } v = x < x_1 \text{.} \)

The vector sum \( u + v \) is parallel to the normal vector of the secondary mirror. Geometrically we can say: slope of the tangent line to the surface at the point \( x = k \) equals the negative reciprocal of the slope of the normal to the surface at the same point.

That is;

\[ n(-1)/n(1) = k \]

We then have \( f(x_1) = 2x_1(x_1^2 + 1)^{1/2} \text{.} \)

Integrating both sides gives the analytical solution:

\[ f(x_1) = x_1^2 + 1/2(x_1^2 + 1)^{1/2} + 1/2(x_1^2(4x_1^2 + 1) + 1) \text{.} \]

Figure 3: Simulation of the double mirror system. For visualization purposes, both mirrors appear in solid colors.

Figure 4: The sideview of the catadioptric sensor with primary mirror a cone of slope 1, and its corresponding dual. Both mirrors are in solid colors for easy visualization.

Figure 5: A 3D degree conical primary mirror and its corresponding dual. For visualization purposes, both mirrors appear in solid colors.

Figure 6: Right Angle Mirror Under Perspective Projection

RIGHT ANGLE MIRROR UNDER ORTHOGRAPHIC PROJECTION

Let us define vectors u and v;

\[ u = 1/2x_1^{1/2}, -1/2x_1^{-1/2} \text{ and } v = a x_1, a > 0 \text{.} \]

By using the same method as the orthographic case, which is after normalizing, \( v \rightarrow \infty \) is in the normal direction of the secondary mirror so its negative reciprocal is the slope of the tangent line to the secondary mirror. Hence, doing the necessary simplifications and substitutions, we obtain a nonlinear differential equation whose numerical solution gives us the cross section of the secondary mirror.

The cross section of the dual mirror can be seen in Figure 5.

Figure 7: 3D Degree Conical Primary Mirror Under Orthographic Projection

CONCLUSION AND FUTURE WORK

In this work, we have exhibited a catadioptric sensor design which enables a normal camera an ultra-wide field of view with no distortion. These sensors are based on a family of double-mirrors with conical primary mirrors derived as solutions of nonlinear ODEs which describe how a plane perpendicular to the optical axis of the system is imaged on the film. The images obtained require no digital unwarping.

In our future work we will;

1. Investigate the general conics as primary mirrors.
2. Implement an actual system.
3. Perform full error analysis.

Figure 8: Sideview of the 3D degree conical primary mirror and its corresponding dual. For visualization purposes, both mirrors appear in solid colors.

Figure 9: This is a PO-V-Ray simulation of the 3D degree conical primary mirror system in the test room. It can be seen how the system scales the viewpoints and there is no distortion.

Figure 10: This is a PO-V-Ray simulation of the 45-degree conical primary mirror system under perspective projection in the test room.