Research Statement

My research interests are in discrete mathematics, including probabilistic methods, asymptotic enumeration, and some topics in number theory. My CV has the required list of publications and presentations, together with links to the papers. It is available online at http://www.pages.drexel.edu/∼schmutze/cv.html, and also at http://www.math.drexel.edu/∼eschmutz/cv.html. This research statement briefly describes the methods used in the papers.

1. Asymptotic analysis:

   In most of my papers, there is a sequence $\langle a_n \rangle_{n=1}^{\infty}$ of numbers that arise naturally in a discrete math setting, and the goal is to estimate the size of a_n for very large values of n. In some cases, one can use Cauchy's Integral Formula to represent the numbers, and derive estimates using the saddle point method, singularity analysis, or other methods based on the approximate evaluation of contour integrals. Most of my papers with Bill Goh use these techniques. The paper on recursive trees [6] was particularly challenging, and it required a variety of tools from complex asymptotics.

2. Probabilistic Methods:

   My papers with Dalal, Hansen, Huang,Perkovic, and Sheng all use probabilistic methods. We used standard first moment arguments, concentration inequalities, etc. in many places. Random geometric graphs were used to analyze algorithms in [17],[18], and implicitly in [19]. In [15] and [20], the flow goes the other way: some (otherwise uninteresting) algorithms are used as analytical devices to prove theorems about random structures. In [8] and [9] respectively, the continuity theorem for moment generating functions was used to prove central limit theorems for the number of part sizes in a random partition and for the number of irreducible factors in the characteristic polynomial of a random matrix in $GL(n, q)$. The second of these two central limit theorems can be deduced from the stronger results in [10].The main theorem in [10] is a matrix analog of De Laurentis and Pittel's theorem on random permutations[3]: the empirical distribution function of "log-degrees"of irreducible factors converges weakly to the Brownian bridge process. Our proof was not easy (convergence of finite dimensional distributions, plus tightness via a fourth moment inequality), but it was it was similar to previous work. In [16], we make a very low-level comparison between random degree n polynomials in $\mathbb{F}_q[x]$, and the characteristic polynomials of random matrices in $GL(n, q)$. The proof uses a complicated multivariate generating function identity of Kung and Stong(which is in turn based on a known formula for the size of each conjugacy class).

3. Number Theory:

   There are many questions about the asymptotic distribution mod 1 of sequences that arise naturally in combinatorics because the numbers of interest are required to be integers.. This happens in both papers on set partitions [11],[12] , and in the three papers on maximum vertex degrees for random trees [6], [2],[13]. The paper [14] was the outcome of outcome of our first efforts to learn about connections between dynamical systems and number theory.

   There is a single paper on diophantine approximation[24].

   Many authors (e.g. Knopfmacher[21],Flajolet-Soria[5], Tavaré et.al[1], Nijenhuis-Wilf[22],[25]) have compared the role components in combinatorial structures to that of the prime numbers in number theory. Although the asymptotic analyses are quite different, both central limit theorems [8] and [9] were motivated by the Erdős-Kac Theorem . I have not used or proved any general theorems in "abstract analytic number theory". However, under the heading "components", my cv lists papers that are clearly influenced by the components-as-primes point of view.

Carmchael's lambda function was investigated in [4]. The proofs are combinatorial, but rely heavily on known results in "multiplicative number theory."This and five other papers are listed on my cv under Erdős and Turán's designation "statistical group theory". All these papers concern the least common multiple of a set of dependent random variables. The settings are quite varied, so it is surprising to me how much these problems have in common. For example, the partition decomposition in [7] (page 39, par.2) is rather analogous to the factorization of characteristic polynomials in [23] (section 3, page 502).

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