Total Magnification and Magnification Centroid Due to Strongly Naked Singularity Lensing

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Abstract

The supermassive galactic center of the Milky Way galaxy is modeled as a strongly naked singularity (SNS) described by the Janis-Newman-Winicour (JNW) metric for an arbitrary mass and scalar charge parameter. The galactic center serving as a gravitational lens gives rise to a series of images on the opposite side of the source from the optical axis and a single image on the opposite side of the source from the optical axis. We compute magnification centroid and magnification centroid shift for various angular source positions and magnifications. We consider different magnification values for different angular source positions. The magnitude of magnification centroid shift increases with increasing magnification and the magnification centroid for different angular source positions is found to approach zero. Total magnification decreases to a limit of 1.

Introduction

The most general static and spherically symmetric solution to the Einstein massless scalar field equations initially obtained by Janis, Newman and Winicour (characterized by constant and real parameters, the mass M and scalar charge q) is expressed by the line-element

\[ ds^2 = -e^{2\varphi}dt^2 + e^{-2\varphi}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \]

\[ \varphi = \varphi_0 - \nu \frac{r^2}{2} \]

The Janis-Newman-Winicour metric has only one photon sphere situated at the radial distance \( R_\text{ps} = \frac{2M}{\nu} \).

Defining the deflection angle \( \beta \) for a light ray as the Janis-Newman-Winicour spacetime is expressed in the form [6a]

\[ d\chi = \frac{1}{\sqrt{-g}} \left( -dt + \frac{\partial\chi}{\partial t} dt \right) + \frac{1}{\sqrt{g}} \left( -dr + \frac{\partial\chi}{\partial r} dr \right) + \frac{1}{r} \left( -d\theta + \frac{\partial\chi}{\partial \theta} d\theta \right) + \frac{1}{r\sin\theta} \left( -d\phi + \frac{\partial\chi}{\partial \phi} d\phi \right) \]

Magnification Centroid & Total Magnification Equations

The magnification centroid of images is expressed by

\[ M = \frac{1}{\sin\beta} \frac{\partial\varphi}{\partial t} \]

where angles measured clockwise to the optic axis are positive and angles counterclockwise are negative.

SNS Lensing for Increasing Values of \( \beta \)

For increasing values of \( \beta \), the total magnification decreases to a limit of 1.

Results - Graphical Analysis

- The magnification centroid and magnification centroid shift are plotted against angular source positions for \( \beta = 0 \) and \( \beta = 1 \).

Discussion

For the purposes of this study, we modeled the supermassive galactic center as a strongly naked singularity (not covered inside any photon sphere). Virbhadra and Keeton [3] studied strongly naked singularity lensing for \( \nu = 1 \) (Schwarzschild Black Hole). We examined \( \nu = 0 \) case, where scalar charge to mass ratio is higher than that considered by Virbhadra and Keeton. Magnification centroid, magnification centroid shift, and total magnification curves are qualitatively similar to their Schwarzschild Black Hole. Due to a higher value of scalar charge to mass ratio, for a fixed value of \( \beta \), magnification centroid will be larger and total magnification will be higher than values found in Schwarzschild black hole lensing. When images are not resolved, total magnification and magnification centroid contribute greatly to our study of lensing. Observations of these results would improve the General Cosmology Hypothesis and would put the way to develop quantum theory of gravity, as very strong gravitational fields will be accessible to observation.

References


Acknowledgements

We would like to thank our mentor, K.S. Virbhadra, for his unflagging dedication and support. His enthusiasm for the subject motivated us to reach new level of understanding and intelligence in our studies. We would like to thank the College of New Jersey for the financial support provided for this project.

Vibrha-Erris Lens Equation

The Virbhadra-Erris gravitational lens equation [7] which captures not only large deflection angles but also large bending angles of light, is given by

\[ D_s \sin \delta_s = D_l \sin \delta_l \]

where \( D_s = \frac{D_l \sin \beta}{\sin \delta} \).

Deflection Angle & Impact Parameter Equations

Virbhadra et al. [6] described the line element as

\[ ds^2 = -e^{2\varphi}dt^2 + e^{-2\varphi}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \]

where \( e^{\varphi} = 1 + \frac{\nu}{2} r^2 \).

They calculated the deflection angle \( \beta \) and impact parameter \( P_\text{imp} \) for a light ray with the closest distance of approach \( r_\text{c} \), which are expressed as

\[ \beta = 2\pi \frac{r_\text{c}^2}{\sqrt{1 - \frac{2M}{r_\text{c}}} - \nu \frac{r_\text{c}^2}{2}} \]

\[ P_\text{imp} = r_\text{c} \sqrt{1 - \frac{2M}{r_\text{c}}} \]

For a Schwarzschild strongly naked singularity:

\[ r_\text{c} = \frac{2M}{\nu} \]

\[ \beta = 2\pi \frac{\nu}{2} \]

\[ P_\text{imp} = \frac{2M}{\sqrt{\nu}} \]

References