Homework #2

These problems are due on Thursday, October 6.

1. Write down the negation of each of the following statements in clear and concise English. Do not use the expression “It is not the case that” in your answers.
   
   (a) $x$ is a real number and $x^2 + 1 = 0$.
   (b) Every integer is divisible by a prime.
   (c) For every real number $x$, there exists an integer $n$ such that $n > x$.
   (d) Every Canadian is a fan of the Toronto Maple Leafs or the Montreal Canadiens.
   (e) $-2 < x < 2$.

2. Exhibit a counterexample to each of the following statements:
   
   (a) $a$ and $b$ are integers and $ab = 1 \rightarrow a = b = 1$.
   (b) If the average of four different integers is 10, at least one of the integers must be greater than 12.
   (c) The product of two irrational numbers is irrational.
   (d) $x - y = y - x$ for all real numbers $x$ and $y$.

3. (a) Prove that $n$ an even integer $\rightarrow n^2 + 3n$ is an even integer.
   (b) What is the converse of the implication in (a)? Is the converse true or false? Justify your answer.

4. (a) Prove (using truth tables) the following tautology:

$$[\neg (p \rightarrow q)] \leftrightarrow (p \land \neg q).$$

   (b) Use 4(a) to negate the following implications:
   
   i. If $x^2 = 4$, then $x = 2$.
   ii. If $x \leq 4$, then $2x \leq 8$.

5. (a) Give a direct proof of the fact that $a^2 - 5a + 6$ is even for any integer $a$.
   (b) Prove that $\sqrt{3}$ is irrational. (Hint: Use an indirect proof.)