Homework #3

These problems are due on Thursday, October 13.

1. (text p.16 #17) Provide a direct proof that $n^2 - n + 5$ is odd for all integers $n$.

2. Closely allied to the Pythagorean theorem is the problem of finding integers $a, b, c$ representing the legs and hypotenuse of a right triangles. You probably know of a few of them, such as 3, 4, 5 and 5, 12, 13 but in fact there are infinitely many such Pythagorean triples representing nonsimilar right triangles. Show that for any odd integer $m$, the three numbers $m, \frac{m^2-1}{2}$ and $\frac{m^2+1}{2}$ yield a Pythagorean triple. (Note that this is not a complete list-there are infinitely many more nonsimilar Pythagorean triples.)

3. Prove that the equation $x^5 + 62x^4 + 17x^3 + 3x + 11 = 0$ has no positive real solutions.

4. Consider the statement : If $n$ is prime, then $2^n + 1$ is prime. Either prove that this is true or give a counterexample to show that this is false.

5. Prove that if $p$ is an odd integer, then $x^2 + x - p = 0$ has no integer solution.

6. (text p.30 #6) Use basic logical equivalences given in section 1.2 to prove that the statements $(p \land \neg q) \rightarrow q$ and $(p \land \neg q) \rightarrow \neg p$ are logically equivalent. What simpler statement is logically equivalent to both of them?

7. (text p.35 #5 (c), (h), (m)) Determine the validity of each of the following arguments. If the argument is one of those listed in the text, then name it.
   
   If I stay up late at night, then I will be tired in the morning.
   
   (a) I am not tired this morning.
       I did not stay up late at night.
   
   (b) If I earn lots of money, then I pay taxes.
       If I work hard, then I pay high taxes.
   
   (c) Either I don’t study or I pass mathematics.
       If I don’t graduate, then I didn’t pass mathematics.
       If I like mathematics, then I will study.
       If I like mathematics, then I will graduate.