1. (a) Write a MATLAB function implementing a predictor-corrector algorithm based on a three-step Adams-Bashforth and a two-step Adams-Moulton methods:

\[ w_{n+1}^{(0)} = w_n + \frac{h}{12} \left\{ 23f(t_n, w_n) - 16f(t_{n-1}, w_{n-1}) + 5f(t_{n-2}, w_{n-2}) \right\}, \]

\[ w_{n+1}^{(1)} = w_n + \frac{h}{12} \left\{ 5f(t_{n+1}, w_{n+1}) + 8f(t_n, w_n) - f(t_{n-1}, w_{n-1}) \right\}, \]

(b) Use (a) to solve numerically the following initial-value problem:

\[ y' = \frac{1}{1+t^2} - 2y^2, \quad y(0) = 0, \quad t \in [0,10], \]

which has the exact solution \( y(t) = \frac{t}{1+t^2} \).

(c) On the same plot, present the exact and approximate solutions obtained using steps: \( h = 0.2, \ h = 0.1, \) and \( h = 0.05 \).

(d) For \( t = 10 \), plot the error between the exact and approximate solutions as a function of \( h, E(h) \). Verify that \( E(h) = O(h^3) \).

2. Consider the initial-value problem

\[ y' = 3t^2y, \quad y(0) = 1. \]

The exact solutions of (1) is \( y(t) = \exp(t^3) \).

(a) Write a MATLAB code to solve (1) on \([0,10]\), using Heun’s method with the following step sizes: \( h = 0.1 \) and \( 2h \). Denote the corresponding approximate solutions by \( w_h \) and \( w_{2h} \) respectively.

(b) Use Richardson’s extrapolation to improve the accuracy of \( w_h \). Denote this solution by \( \tilde{w}_h \).

(c) On the same figure, plot the graphs of \( y, w_h, w_{2h} \), and \( \tilde{w}_h \).

(d) On another figure, plot the errors of approximate solutions: \( e_h = y - w_h, \ e_{2h} = y - w_{2h}, \)

\( \tilde{e}_h = y - \tilde{w}_h \) (use different colors). What is the most accurate solution?