Homework #8

#21
Evaluate \( \int_C z e^{z^2} \, dz \), \( C \) from 1 along the axes to 1.

Note that \( z e^{z^2} \) is analytic everywhere, so the method applies:

\[
\int_C z e^{z^2} \, dz = \frac{1}{2} \left. e^{z^2} \right|_1^i = \frac{1}{2} (e^1 - e^i) = -\left( \frac{e^i - e}{2} \right) = -\sinh 1
\]

#26
Evaluate \( \oint_C \left( \frac{3}{z-1} - \frac{4}{(z-i)^2} \right) \, dz \), \( C \) with \( |z-1| = 5 \) clockwise.

\( f(z) \) has singularity at \( z = i \).

Parametrize \( C : z(t) = 1 + 5 \, e^{-it} \), \( 0 \leq t \leq 2\pi \)

Then \( \, dz = -5i \, e^{-it} \, dt \)

Therefore:

\[
\oint_C \left( \frac{3}{z-1} - \frac{4}{(z-i)^2} \right) \, dz = \int_0^{2\pi} \left( \frac{3}{5 \, e^{-it}} - \frac{4}{25 \, e^{-2it}} \right) (-5i \, e^{-it}) \, dt
\]

\[
= \int_0^{2\pi} \left( -\frac{3i}{5} + \frac{20i}{25} e^{it} \right) \, dt = \left[ -\frac{3i}{5} \frac{1}{i} + \frac{20i}{25} e^{it} \right]_0^{2\pi} = -6\pi i + \frac{6\pi}{5} i \left( e^{2\pi i} - e^0 \right) = -6\pi i
\]
11.3.2 Integrate $f(z) = \frac{1}{2z-1}$ around the unit circle. $C: |z| = 1$

\[ \oint_C \frac{1}{2z-1} \, dz = \frac{1}{2} \oint_C \frac{1}{z - \frac{1}{2}} \, dz \]

$z = \frac{1}{2}$ is singularity, enclosed by $C$, so Cauchy's Thm. doesn't apply.

Then parameterize $C$, by Principle of Deformation of path: $C: z(t) = \frac{1}{2} + e^{it} \Rightarrow dz = i e^{it} \, dt$

\[ z = \frac{i}{2} \int_0^{2\pi} \, e^{it} \, dt = \frac{i}{2} \cdot \frac{e^{i2\pi} - e^i0}{i} = 2\pi i \]

11.3.2 Integrate $C: |z| = \pi$. $z = 3i$ is the singularity which is enclosed by $C$.

So, Cauchy's Thm. doesn't apply.

Parameterize $C$, by deformation of path:

$z(t) = 3i + \pi e^{it} \quad 0 \leq t \leq 2\pi \Rightarrow dz = \pi i e^{it} \, dt$

\[ \int_0^{2\pi} \frac{\pi i e^{it}}{\pi} \, dt = 2\pi i \]