

Mathematical Neuroscience

Homework Project 1. Due April 6 in class.

The material for this part of the course is covered in §§ 2.1, 2.2, 2.4, and 2.5 in the course notes by P. Eckhoff and P. Holmes and in the handouts ‘Elements of the theory of differential equations’ and ‘Elements of the bifurcation theory’.

Problems:

1. For each of the following systems, find the fixed points. Linearize the system about these points. Classify the fixed points (saddle, stable/unstable node, etc). Sketch the neighboring trajectories and try to fill in the rest of the phase portrait.

a.

$$\begin{aligned}\dot{x} &= -x + x^3 \\ \dot{y} &= -2y\end{aligned}$$

b.

$$\begin{aligned}\dot{x} &= x(3 - x - 2y) \\ \dot{y} &= y(2 - x - y)\end{aligned}$$

c.

$$\begin{aligned}\dot{x} &= y \\ \dot{y} &= x - x^3\end{aligned}$$

2. **Problem 2** is posted on the web (below Topics). Follow the steps described in this problem. After that, modify the parameters of the model and repeat all the steps. In your report, include your codes and the representative plots.
3. Write a matlab code for plotting the phase portrait for the equation of pendulum:

$$\begin{cases} \dot{x} = y, \\ \dot{y} = -\sin x. \end{cases}$$

Include the code and the phase plane plot.