Numerical Analysis- MATH 300 Project # 1
Due: Wednesday, October 20, 2010 by beginning of class time. All work is expected to be independent! Hand in all Matlab codes, final output, and any supplementary calculations and discussion.


   (a) Write a program to perform Newton’s method on $f(x) = x^3 - 25$ with $p_0 = 3$ to get an approximation to $\sqrt[3]{25}$ that is accurate to within $10^{-15}$. Save all of the iterates.

   (b) Compute the error and the number of significant digits of accuracy for each iterate.

   (c) Beginning with the interval $[2, 3]$, how many iterates would bisection have needed to get error this small?

2. Zeros of Polynomials. The polynomial

   
   $f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$

   has 2 real zeros, one in $[-1, 0]$ and one in $[0, 1]$.

   (a) Try to find both of these zeros to within $10^{-6}$ using Newton’s method.

   (b) Try to find both of these zeros to within $10^{-6}$ using the Secant method.

   (c) Try to find all four (two real and two complex) zeros by using Müller’s method.

3. Fixed Point Iteration. The following four methods are proposed to compute $7^{1/5}$. Starting with $p_0 = 1$, rank the four methods in order based on their apparent speed of convergence. In each case, explain the behavior you see.

   (a) $p_n = p_{n-1} \left( 1 + \frac{7-p_{n-1}^5}{p_{n-1}^5} \right)^3$

   (b) $p_n = p_{n-1} - \frac{p_{n-1}^5 - 7}{p_{n-1}^5}$

   (c) $p_n = p_{n-1} - \frac{p_{n-1}^5 - 7}{5p_{n-1}^4}$

   (d) $p_n = p_{n-1} - \frac{p_{n-1}^5 - 7}{12}$