MATH 102 Exam 3 Review

1. Express \( \frac{dy}{dx} \) in terms of \( x \):
   (a) \( y = u^3 + 5; u = 3 + e^{2x} \)
   (b) \( y = e^{3w}; w = -3u + 4; u = \ln(2x) \)

2. Find the relative rate of change for \( f(x) = 3x + x \ln 5x \)

3. Evaluate:
   (a) \( \int \sqrt{2x + 1} \, dx \) (b) \( \int \left( \frac{3}{x^2} - \frac{4}{x} \right) \, dx \) (c) \( \int 4e^{3x} \, dx \) (d) \( \int xe^{x^2 + 3} \, dx \)
   (e) \( \int \frac{1 - y^2}{\sqrt{y}} \, dy \) (f) \( \int \frac{x}{(2x^2 - 5)^3} \, dx \) (g) \( \int \left[ 5x^2 - 2x^{-\frac{1}{2}} \right] \, dx \) (h) \( \int \left[ 32(x^2 + 1)^7 \right] \, x \, dx \)
   (i) \( \int_{-1}^{1} \frac{x}{x^2 + 1} \, dx \) (j) \( \int_{0}^{e} \left( \ln(t) \right)^2 \, t \, dt \)

4. What is the average value of \( f(x) = x^3 - 4x \) over \([0,4]?)

5. A company estimates that oil will be pumped from a producing field at a rate
   \( r(t) = \frac{120t}{t^2 + 1} + 3 \), \( 0 \leq t \leq 20 \), where \( r \) is the rate of production (in thousands of barrels per year) \( t \) years after pumping begins. Approximately how many barrels of oil will the field produce during the first 5 years of production? The second 5 years of production?

6. Use 4 left rectangles to estimate the area under the curve \( y = 2x + 2 \) from \( x = 1 \) to \( x = 3 \).

7. If \( \frac{dy}{du} = 5 \), \( \frac{du}{dv} = -2 \), \( \frac{dv}{dx} = 3 \), what is \( \frac{dy}{dx} \)?

8. Find \( x(t) \) that satisfies: \( \frac{dx}{dt} = \frac{3\sqrt{t} - t}{2\sqrt{t}} \), \( x(27) = -100 \)

9. (a) \( \frac{d}{dx} \left( \int e^x \, dx \right) = \)

   (b) \( \int \frac{d}{dx} \left( \frac{x + 2}{x^5 + 7x + 5} \right) \, dx = \)
1. (a) \(6(3 + e^{2x})^2e^{2x}\)  \(\frac{-9e^{12}}{2^9x^{10}}\)  
(b) \(\frac{4 + \ln 5x}{3x + x \ln 5x}\)

2. \(\frac{2}{3}(2x + 1)^\frac{3}{2} + c\)  \(\frac{-3}{x} - 4 \ln x + c\)  \(\frac{4}{3}e^{3x} + c\)

3. (a) \(\frac{2}{3}(2x + 1)^\frac{3}{2} + c\)  \(\frac{-3}{x} - 4 \ln x + c\)  \(\frac{4}{3}e^{3x} + c\)

(d) \(\frac{1}{2}e^{x^3} + c\)  
(e) \(\frac{1}{2}y - \frac{2}{5}y^5 + c\)  
(f) \(\frac{1}{28(2x^2 - 5)} + c\)

4. 8

5. 60 \(\ln (26)\) thousands of barrels (about 195486 barrels)
60 \(\ln(101/26)\) thousands of barrels (about 81,421 barrels)

6. 11 7. -30 8. \(x(t) = t - \frac{3}{5}t^3 + \frac{94}{5}\)  
9. (a) \(e^{x^2}\)  
(b) \(\frac{x + 2}{x^2 + 7x + 5} + c\)