1. Consider the graph of \( f(x) \) below:

\[
\begin{array}{c}
\text{(a) } \lim_{x \to 0^-} f(x) = \\
\text{(b) } \lim_{x \to 0^+} f(x) = \\
\text{(c) } \lim_{x \to 1^-} f(x) = \\
\text{(d) } \lim_{x \to 1^+} f(x) = \\
\text{(e) } \lim_{x \to 1} f(x) = \\
\text{(f) } \lim_{x \to 1} f(x) = \\
\text{(g) } \lim_{x \to 2} f(x) = \\
\text{(h) For what values of } x \text{ is } f(x) \text{ discontinuous (not continuous)?}
\end{array}
\]

2. Let \( f(x) = \begin{cases} x^2 - 3 & \text{if } x \leq 1 \\
2x & \text{if } x > 1 \end{cases} \)

(a) \( \lim_{x \to 1^-} f(x) = \)  
(b) \( \lim_{x \to 1^+} f(x) = \)  
(c) \( \lim_{x \to 1} f(x) = \)  
(d) Is \( f(x) \) continuous at 1?

3. Let \( f(x) = \begin{cases} x^2 - 10 & \text{if } x \leq 4 \\
2x - 2 & \text{if } x > 4 \end{cases} \)

(a) \( \lim_{x \to 4^-} f(x) = \)  
(b) \( \lim_{x \to 4^+} f(x) = \)  
(c) \( \lim_{x \to 4} f(x) = \)  
(d) Is \( f(x) \) continuous at 4?

4. Compute the following limits (if they do not exist, state that this is the case):

(a) \( \lim_{x \to 1} \frac{x^2 + x - 2}{2x - 3} = \)  
(b) \( \lim_{x \to 1} \frac{x^2 + x - 2}{x^3 - 3x + 2} = \)  
(c) \( \lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - 3x + 2} = \)  
(d) \( \lim_{x \to 3} \frac{3x + 4}{x^2 - 3x + 2} = \)

5. (a) Determine where the function \( f(x) = \frac{(3x - 2)}{(x - 2)(x - 3)} \) is continuous.

(b) Repeat part (a) for the function \( g(x) = \sqrt{9 - x^2} \)
6. Solve the inequalities below for $x$ and express your answers in interval notation.

(a) $x^2 + 7x + 10 < 0$ \hspace{1cm} (b) $\frac{2x + 5}{(x-4)(x-3)} > 0$

7. Use the definition of derivative (four step process) to find $f'(x)$ (no other method will be accepted):

(a) $f(x) = 2 - \frac{1}{x}$ \hspace{1cm} (b) $f(x) = 3x^2 - 5x$

8. A person $x$ inches tall has a pulse rate of $y$ beats per minute as given by $y = 590x^{-1/2}$, $30 \leq x \leq 75$. What is the average rate of change in $y$ when $x$ is between 36 and 49? What is the instantaneous rate of change of the pulse rate when the height is 49 inches?

9. Compute the following derivatives:

(a) $y = (3x^2 - 2x)(3x^2 - 4x + 5)^7$ \hspace{1cm} (b) $f(x) = \frac{2x^4 - 3x}{x^3 - 2x + 3}$ \hspace{1cm} (c) $g(x) = x\sqrt{3x + 4}$

(d) $g(x) = \frac{x}{\sqrt{4x^2 + 5}}$ \hspace{1cm} (e) $y = \frac{1}{\sqrt{2x}} + 2\sqrt{x}$

10. Find the equation of the line tangent to the curve $f(x) = \sqrt{3x + 1}$ at $x=1$.

11. Find the value(s) of $x$ where the tangent line is horizontal

(a) $f(x) = (x + 3)(x^2 - 45)$ \hspace{1cm} (b) $f(x) = \frac{x}{x^2 + 4}$

12. A sewage treatment plant disposes of its effluent through a pipeline that extends 1 mile toward the center of a large lake. The concentration of the effluent $C(x)$, in parts per million, $x$ meters from the end of the pipe is given by $C(x) = 500(x + 1)^{-2}$. Find $C'(8)$ and interpret.

Solutions:

1. (a) 3 (b) -1 (c) DNE (d) 1 (e) 1 (f) 1 (g) 2 (h) $x = 0,1$

2. (a) 2 (b) -2 (c) DNE (d) No 3. (a) 6 (b) 6 (c) 6 (d) Yes

4. (a) 0 (b) 5 (c) -3 (d) DNE 5. (a) $(-\infty, 2) \cup (2,3) \cup (3,\infty)$ \hspace{1cm} (b) $[-3,3]$

6. (a) $(-5, -2)$ \hspace{1cm} (b) $\left(-\frac{5}{2}, 3\right) \cup (4,\infty)$
7(a) For \( f(x) = 2 - \frac{1}{x} \)

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{2 - \frac{1}{x+h} - \left(2 - \frac{1}{x}\right)}{h}
\]

\[
= \lim_{h \to 0} \frac{2 - \frac{1}{x+h} - 2 + \frac{1}{x}}{h} = \lim_{h \to 0} \frac{1}{x+h} - \frac{1}{x} = \lim_{x \to 0} \frac{x+h-x}{x(x+h)}
\]

\[
= \lim_{h \to 0} \frac{h}{hx(x+h)} = \lim_{h \to 0} \frac{1}{x(x+h)} = \frac{1}{x^2}
\]

(b) For \( f(x) = 3x^2 - 5x \)

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{3(x+h)^2 - 5(x+h) - (3x^2 - 5x)}{h}
\]

\[
= \lim_{h \to 0} \frac{3(x^2 + 2xh + h^2) - 5x - 5h - 3x^2 + 5x}{h} = \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 - 5h - 3x^2}{h}
\]

\[
= \lim_{h \to 0} \frac{6xh + 3h^2 - 5h}{h} = \lim_{h \to 0} h(6x + 3h - 5) = \lim_{h \to 0} (6x + 3h - 5) = 6x - 5
\]

8. \(-250/(7)(7)(7) = -.7289 \text{ beats/min/inch}\)

9. (a) \( y' = \left(3x^5 - 4x + 5\right)^6 \left(7 \left(3x^2 - 2x\right) (15x^4 - 4) + \left(3x^5 - 4x + 5\right) (6x + 2)\right) \)

(b) \( f''(x) = \frac{(x^3 - 2x + 3) (8x^3 - 3) - (2x^4 - 3x) (3x^2 - 2)}{(x^3 - 2x + 3)^2} \)

(c) \( g'(x) = \frac{3x}{2\sqrt{3x + 4}} + \sqrt{3x + 4} \)  \( g'(x) = \frac{4}{3} x^2 + 5 \)

(d) \( g'(x) = \frac{4}{3} x^2 + 5 \)

(e) \( -\frac{1}{\sqrt{8x^3}} + \frac{1}{\sqrt{x}} \)

10. \( y = \frac{1}{4} x + 5/4 \)  11. (a) \( x = -5, 3 \)  (b) \( x = 2, -2 \)

12. \( C'(8) = -1000 / 729 = \text{ rate at which effluent is decreasing 8 meters from end of pipe} \)