MATH 102 Review sheet Exam 2

1. Let \( f(x) = x^3 - 6x + 20 \)

(a) \( f' = \) _____________________

(b) \( f'' = \) ___________________

(c) Make a sign chart for \( f' \) and \( f'' \).

(d) For what values of \( x \) is \( y \) increasing? decreasing?

(e) Identify all relative maxima and minima or state that there are none (Give (x,y) coordinates)

(f) For what values of \( x \) is \( y \) concave up? concave down?

(g) Give the (x,y) coordinates of any inflection points or else state that there are none.

(h) Sketch the function labeling all relative max-min and inflection pts

2. For the following functions, determine where they are increasing, decreasing, and any local extrema:

(a) \( f(x) = 3 - \frac{4}{x} - \frac{2}{x^2} \)  
(b) \( f(x) = x^4 (x - 3)^6 \)  
(c) \( f(x) = (x - 2)e^{4x} \)  
(d) \( f(x) = \frac{\ln(x)}{x} \)

3. Find the absolute maximum and minimum for the function

\[ f(x) = x^5 - 5x^4 + 7 \] \text{ on } [-1,1], [-1,5].

4. A commercial pear grower must decide when is the optimum time to pick fruit. If pears are picked now, they will bring 30 cents / pound, with each tree yielding an average of 60 pounds of pears. If the average yield per tree increases 6 pounds per tree per week for the next 4 weeks, but the price drops 2 cents per pound per week, when should the pears be picked to realize the maximum return per tree? What is the maximum return per tree?

5. A recent study of the exercise habits of 17,000 college alumni found that the death rate (deaths per 10,000 person-years) was approximately \( R(x) = 5x^3 - 35x + 104 \), where \( x \) is the weekly amount of exercise in thousands of calories \( (0 \leq x \leq 4) \). Find the exercise level that minimizes the death rate.
6. Find derivatives for the following functions:
(a) \( f(x) = x^3e^{4x} \)  
(b) \( g(x) = \ln(e^x + 3x) \)  
(c) \( h(x) = \left(\ln(2x + 5)\right)^3 \)

(d) \( y = \frac{e^{5x}}{x^3} \)  
(e) \( f(x) = \ln\left((2x + 5)^3\right) \)  
(f) \( g(x) = 2x^4 \)

7. Same as problem 1 for (a) \( f(x) = 2 - 3e^{-2x} \) and (b) \( g(x) = \ln(x^2 + 4) \)

SOME SOLUTIONS
1(a) \( f'(x) = 3x^2 - 6 \)  
(b) \( f''(x) = 6x \)  
(d) incr\([-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)\]  
edecr \((-\sqrt{2}, \sqrt{2})\)  
(e) rel max at \(-\sqrt{2}\), rel min at \(\sqrt{2}\)  
(f) concave up \((0, \infty)\)  
conceve down \((-\infty, 0)\)  
(g) \((0, 0)\) inflection point

2(a) incr \((-\infty, -1) \cup (0, \infty)\)  
decr \((-1, 0)\) local max at \(x = -1\)

(b) incr \((0, 6/5) \cup (3, \infty)\)  
decr \((-\infty, 0) \cup (6/5, 3)\) local min at \(x = 0, 3\) local max at \(x = 6/5\)

(c) incr \((7/4, \infty)\)  
decr \((-\infty, 7/4)\) local min at \(x = 7/4\)

(d) incr \((0, e)\) decr \((e, \infty)\) local max at \(x = e\)

3(a) max value of 7 at \((0, 7)\); min value of 1 at \((-1, 1)\)

(b) max value of 7 at \((0, 7)\) and at \((5, 7)\); min value of -249 at \((4, -249)\)

4. pick in 2.5 weeks for max return per tree of $18.75

5. 3500 calories per week

6 (a) \( f'(x) = e^{4x} \left(4x^3 + 3x^2\right) \)  
(b) \( g'(x) = \frac{e^x + 3}{e^x + 3x} \)  
(c) \( h'(x) = 3\left(\ln(2x + 5)\right)^2 \frac{2}{2x + 5} \)

(d) \( y' = \frac{e^{5x}(5x - 2)}{x^3} \)  
(e) \( f''(x) = \frac{6}{2x + 5} \)  
(f) \( g'(x) = 2^x(1 + x \ln 2) \)

7(a) \( f'(x) = 6e^{-2x} \)  
(b) \( f''(x) = -12e^{-2x} \)  
(d) \( f \) is always increasing \((-\infty, \infty)\)

(e) There are no local extrema  
(f) \( f \) is always concave down  
(g) There are no inflection points.

(b) \( a \) g'(x) = \frac{2x}{x^2 + 4}  
(b) \( b \) g''(x) = \frac{2(4 - x^2)}{x^2 + 4}  
(d) increasing \((0, \infty)\); decr \((-\infty, 0)\)

(e) local min at \((0, \ln 4)\)  
(f) concave up \((-2, 2)\); concave down \((-\infty, -2) \cup (2, \infty)\)

(g) inflection points: \((2, \ln 8), (-2, \ln 8)\)