Math 180, Spring 2006
Quiz 4
With Solutions

Your Name:

1 Problem 1

Give numeric values (no factorials!) for the following quantities:

a. \(5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120\)

b. \(\frac{100!}{99!} = \frac{1 \cdot 2 \cdot \ldots \cdot 99 \cdot 100}{1 \cdot 2 \cdot \ldots \cdot 99} = 100\)

c. \(\binom{6}{3} = \frac{6!}{3!(6-3)!} = \frac{6!}{3!3!} = 20\)

d. \(\binom{1000}{2} = \frac{1000!}{2!(1000-2)!} = \frac{1000!}{2!998!} = \frac{999 \cdot 1000}{2} = 499500\)

e. \(\binom{1000}{1000} = \frac{1000!}{1000!0!} = 1\)

2 Problem 2

Factorials and "choose"-notation OK!

a. How many ways are there to distribute 5 different books among 5 different people?

\(5! = 120\)

b. What’s the answer to the question above if 3 of those books are the same?

Answer: Choose the two people who will get the different books first: \(\binom{5}{2}\). Subsequently distribute the 2 different books between them: 2!.

\(\binom{5}{2}2!\)
c. A pizza shop has 5 different toppings. How many different pizzas can they make?

Answer: For each topping, you either have it or not:

\[ 2^5 \]

d. A man, a woman, a boy, a girl, a dog, a cat and a rabbit (that’s 7 objects) are walking down the street. The dog is immediately following the cat. Given that, in how many ways can you order the members of this family?

Answer: Think of the cat-dog as a single object. That leaves 6 objects to rearrange:

\[ 6! \]

e. Same problem, but additionally you know that the cat (in quiz was "dog") is in the very front of the group.

Answer: This leaves 5 object to rearrange:

\[ 5! \]

3 Problem 3

Factorials and "choose"-notation OK!

a. How many 7 digit numbers are there?

Answer: The first digit can be any of 9 digits (that is 1 – 9), the remaining 6 can be any of 10 (that is 0 – 9), so:

\[ 9 \times 10^6 \]

b. How many 7 digit numbers are there that consist only of 3’s?

Answer: There is only one:

\[ 3333333 \]

c. How many 7 digit numbers are there that consist of two 3’s and five 8’s?

Answer: First choose the two slots where the 3’s go, \( \binom{7}{2} \). The rest are occupied by the 8’s so no decisions left to make:

\[ \binom{7}{2} \]

d. How many 7 digit numbers are there that consist of two 3’s, two 8’s, and three 9’s?

Answer: First choose the 2 slots where the 3’s go, \( \binom{7}{2} \). Then, of the remaining 5 slots, choose the two slots where the 8’s would go, \( \binom{5}{2} \). The remaining three slots are occupied by the 9’s so no decisions left to be made:

\[ \binom{7}{2} \binom{5}{2} \]
e. How many 7 digit numbers are there that consist of two 3’s, two 8’s and four 9’s?
Answer: None. That’s 8 digits. (Not a great problem.)
f. How many 7 digit numbers are there that consist of two 3’s, two 8’s and three (in the quiz, it said "four") 0’s?
Answer: A 0 cannot be in the first slot. So you have two choices: either a 3 or an 8 need to go in the first slot. If a 3 is in the first slot, that leaves ONE 3, TWO 8’s and THREE 0’s for the remaining slots, which is \( \binom{6}{1}\binom{5}{2} \) combinations. If an 8 is in the first slot, that leaves TWO 3, ONE 8 and THREE 0’s for the remaining slots, which is \( \binom{6}{2}\binom{4}{1} \) combinations (which is actually the same number as \( \binom{6}{1}\binom{5}{2} \)). The combined answer is
\[
\binom{6}{1}\binom{5}{2} + \binom{6}{2}\binom{4}{1}
\]
g. How many 7 digit numbers are there that consist only of 2’s, 3’s, and 4’s?
Answer: Three choices for each one of the slots so
\[3^7.\]
h. How many 7 digit numbers are there that consist only of 2’s, 3’s, and 0’s?
Answer: Two choices for the first slot (i.e. 2 or 3) and three choices for each remaining slot so
\[2 \cdot 3^6.\]