Homework 3
Due Tuesday, February 14th at 11:59pm

1 Problem 1
Matlab and IEEE
a. What is the lowest positive integer not represented by an IEEE double?
b. What is the largest integer represented by an IEEE double?
c. What is the largest number represented by an IEEE double?
d. What is the smallest number greater than 1 represented by an IEEE double?
e. Are there more IEEE doubles between 1 and 2 or between 2 and 4?
f. Evaluate the following values and understand the results (nothing to submit)
\[2^{54} + 1 - 2^{54} - 2^{-53} + 2^{52} \times 2^{100} \times 2^{-52} + 1 - 2^{100} - 2^{52} \times 2^{1000} - 2^{-52} + 1 - 2^{1000} - 1/\text{inf}; \sin(\text{inf}); \log(\text{inf}); e^{-\text{inf}}; (1 + \text{NaN})/\text{inf};\]
g. Assume that you are not allowed to call functions \(\cosh(x)\), \(\sinh(x)\), or \(\tanh(x)\), but you are allowed to call \(\exp(x)\). Program a function \(\text{tanh300.m}\) which evaluates \(\tanh(x)\), but in such a way that \(\tanh(19)\) is distinguishable from 1 and \(\tanh(1000)\) is not return NaN. Recall that \(\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}\). (However, \(f = \text{inline}('\frac{(\exp(x) - \exp(-x))}{(\exp(x) + \exp(-x))}', 'x')\) is not the right answer!)

2 Problem 2
Matlab/IEEE exercise

2.1 Part a
Write a function \text{num2bin} that converts a number to its IEEE representation:
\[>> \text{num2bin}(7.81) \]
\[01000000001111011011110001010011110101110000101000111101\]
2.2 Part b
Include an additional input parameter that (if "true") will break the string up like this:

```matlab
>> num2bin(7.81, true)
0
10000000001 1111011101011000010100011110110110000101000111101
```

2.3 Part c
Write a function bin2num that would convert the IEEE representation string back to the number.

2.4 Hints
You will need to learn how to deal with strings in Matlab. Start with "help strings". I Can also benefit from the num2hex command (help num2hex).

3 Problem 3
Use interpolation to derive a finite difference estimate for the second derivative \(f''\) for on an irregular stencil: \(x_i - x_{i-1} = h_1, h_2 = x_{i+1} - x_i\), where \(h_1 \neq h_2\)

4 Problem 4
4.1 Part a
Use interpolation to derive a finite difference estimate for the second derivative \(f''\) on a regular 5 point stencil \(x_{i-2}, x_{i-1}, x_i, x_{i+1}, x_{i+2}\).

4.2 Part b
Determine the order of convergence of your estimate.