1 Problem 1

Implement the one dimensional Finite Element Method for the Laplace equation on an irregular grid. The signature of the function should be as follows:

\[
\text{function } [ u \ M \ b ] = \text{fem1D}(x, A, B)
\]

although what you name the function is up to you. The vector \( x \) contains the location of the nodes.

Write two versions of this function (you can give them different names) that handle boundary conditions by a). Lagrange multipliers and b). direct substitution.

Also write and run the following "driver" for your function
\[
\begin{align*}
 & \text{h} = 0.01; \\
 & \text{x} = \text{linspace}(0, 1, \text{h}); \text{ This should produce the same } u, M, \text{ and } b \text{ as in class} \\
 & x = [0 \ \text{rand}(1, \text{ceil}(1/h)) \ 1] ;
\end{align*}
\]
\[
\begin{align*}
 & A = 4; \\
 & B = 7; \\
 & u = \text{fem1D}(x, A, B); \\
 & \text{plot}(x, u, 'ro-')
\end{align*}
\]

Submit all code and the plots.