Accelerated Calculus I — Review Problems

Instructions: Work through all these problems in your group. Please keep the work neat and as clear as possible. The due date is Wednesday in class.

1. Compute each of the following limits, if possible.
   
   (a) \( \lim_{x \to 2} \frac{2^2 - 4x + 4}{x^2 + x - 6} \).
   
   (b) \( \lim_{x \to 0} \frac{\sqrt{x^2 + 4} - 2}{x} \).
   
   (c) \( \lim_{x \to 3^-} \frac{x}{x - 3} \).
   
   (d) \( \lim_{x \to -\infty} (2x^3 - 100x + 5) \).
   
   (e) \( \lim_{x \to \infty} \frac{5x^2 + 7}{3x^2 - x} \).
   
   (f) \( \lim_{x \to 0} \frac{\sin 6x}{x} \).
   
   (g) \( \lim_{t \to 0} \frac{t^2}{1 - \cos^2 t} \).

2. Find all values of \( x \) at which \( f(x) = 3/(5 + 2 \cos x) \) is continuous.

3. Use the Intermediate Value Theorem to show that the equation \( x^3 - 4x + 1 = 0 \) has a solution in the interval \((0, 1)\).

4. Compute the derivative \( f'(x) \) of \( f(x) = x^2 - x \) from its definition as a limit.

5. Do the indicated calculation:
   
   (a) Compute the derivative of \( y = 7x^{-6} - 5\sqrt{x} \).
   
   (b) Compute the derivative of \( y = \frac{4x+1}{x^2+5} \).
   
   (c) Compute the derivative of \( f(x) = (x^2 + 1) \sec x \).
   
   (d) Compute the second derivative of \( \csc x \).

6. Find the equation of the tangent and normal lines to the graph of \( y = \sin x \) at \( x = \pi \).

7. Find all values of \( x \) at which \( f(x) = x^4/4 + x^3/3 - x^2 + 1 \) has a horizontal tangent line.

8. Find the slope of the tangent line to the graph of \( x^3y + xy^3 = 10 \) at the point \((2, 1)\).

9. A 10 foot plank is leaning against a wall. If, at a certain instant, the bottom of the plank is 2 feet from the wall, and is being pushed toward the wall at the rate of 1/2 feet per second, how fast is the angle the plank makes with the ground increasing?
10. Perform the indicated calculation:

(a) Find the derivative of \( y = \ln(x^2 e^x) \).
(b) Find the derivative of \( y = \exp[x^3] \).
(c) Find the derivative of \( y = \arctan(1/x) \).
(d) Find the derivative of \( y = x \arcsin(\tan x) \).
(e) Find the derivative of \( y = (x^2 + 3)^{\ln x} \).
(f) Find the derivative of \( y = \frac{\sin x \cos x \tan^3 x}{\sqrt{x}} \).

11. Compute the indicated limits:

\[ (a) \lim_{x \to \infty} \frac{\ln(\ln x)}{\ln x}, \quad (b) \lim_{x \to 0^+} \csc 7x \sin 3x, \quad (c) \lim_{x \to 1^-} x^{\frac{1}{x-1}}, \quad (d) \lim_{x \to \infty} (\ln x - \ln(1 + x)). \]

12. Find the critical points of \( f(x) = 4x^3 + 6x^2 - 9x \) and use the second derivative test to determine if they are relative maximum or relative minimum points.

13. Let \( f(x) = x^4 + 2x^3 - 1 \). Find all of the intervals on which \( f(x) \) is increasing, decreasing, concave up, concave down, all inflection points, and classify each critical point as a relative maximum, relative minimum, or neither.