1. (a) Find the Maclurin expansion (=Taylor expansion about $x_0 = 0$) for $\sin(3t)$. (b) Use the Lagrange Form of the Remainder to estimate how closely the fifth degree maclurin polynomial $p_5(x)$ approximates $\sin(3t)$ on the interval $[0,0.1]$? (c) Find the value of $a$ so that the first-degree maclurin polynomial $p_1(t)$ approximate $\sin(3t)$ to within $10^{-2}$ on the interval $[0,a]$.

Comments: (a) Recall that $\sin x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$. Set $x = 3t$ to obtain

$$\sin(3t) = \sum_{k=0}^{\infty} (-1)^k \frac{(3t)^{2k+1}}{(2k+1)!} = 3 \sum_{k=0}^{\infty} (-1)^k \frac{9t^{2k+1}}{(2k+1)!}.$$ 

(b) Let $f(t) = \sin 3t$. Then $|f(t) - p_5(t)| = |f^{(6)}(c)t^6/6!|$, where $c$ lies between 0 and $t$. But $f^{(6)}(c) = -3^6 \sin(3c)$. Hence $|f(t) - p_5(t)| \leq 3^6 \sin(0.3)(0.1)^6/6! = 0.2992142093 \times 10^{-6}$.

(c) We note that $a$ must satisfy: $|p_1(t) - \sin(3t)| = |f^{(4)}(c)t^2/2|$, where $c$ lies between 0 and $t$. We require that $t \in [0,a]$ so $|f^{(4)}(c)t^2/2| \leq 3^2 \sin(3a)a^2/2$. We make a crude estimate that $|\sin(3a)| \leq 1$. Hence, $a$ will satisfy $a^2 = (2/9)10^{-2}$ or $a = .04714045208$. If we take $\sin(3a)$ into account, we find that $a = .09085574003$ roughly twice as large as the previous estimate.