1. (a) Let \( f(x) = \sin(x) \). Let \( p_n(x) \) be its \( n \)-th degree Taylor polynomial centered at 0. For \( n = 5 \), find an error bound for \( |f(x) - p_n(x)| \) if \( x \in [-0.2, 0.2] \).

(b) Let \( p_3(x) \) be the third degree Taylor polynomial for \( \cos x \) centered at 0. Find an interval of the form \((-b, b)\) where the error of replacing \( \cos x \) with \( p_3(x) \) is less than \( 1/500 \).

2. (a) Suppose \( p^* \) must approximate \( p \) with relative error at most \( 10^{-4} \). Find the largest interval in which \( p^* \) must lie if \( p = 9000 \).

(b) Use a Taylor series expansion to rewrite the following difference to avoid loss of significance. Please include at least two non-zero terms in the expansion.

\[
F(h) = \exp(h^2) - \cos(h).
\]

(c) Rewrite the difference \( \sqrt{1 + x^2 - x^2} \) algebraically to avoid loss of significance.

3. (a) How many iterations of the Bisection Method is needed to find a solution accurate to within \( 10^{-2} \) for the rootfinding problem \( x^3 + 4x^2 - 10 = 0 \) on \([1, 2]\).

(b) How many iterations of the Bisection Method is needed to find a solution accurate to within \( 10^{-2} \) for the rootfinding problem \( 2x\cos(2x) - (x + 1)^2 = 0 \) on \([-3, -2]\).

4. Use the error estimate for fixed point iteration to determine the number of iterations to guarantee that the approximation is within \( 10^{-2} \) to the fixed point of \( f(x) = (10/(4 + x))^{1/2} \).

5. (a) Find the Maclaurin expansion of \( \cos(t^2) \). Exhibit at least 4 non-zero terms.

(b) Find the Maclaurin expansion of \( \int_0^x \cos(t^2) \, dt \). Exhibit at least 4 non-zero terms.

(c) Use part (b) to find an alternating series for the value of the integral \( \int_0^{0.1} \cos(t^2) \, dt \). Exhibit at least 4 non-zero terms.

(d) Use the Alternating Series Test to find an estimate of the integral \( \int_0^{0.1} \cos(t^2) \, dt \) to within 0.005.

6. (a) Circle the correct choice.

Let \( x_n = 1/n^2 \). Then the sequence converges to 0:

(a) of order 1, (b) of order 2, (c) neither.

(b) Circle the correct choice.

Let \( x_n = 0.5 \times 3^{-n} \). Then the sequence converges to 0:

(a) of order 1, (b) of order 2, (c) neither.

(c) Circle the correct choice.

Let \( f(x) = x\sin(x) \). Then \( f(x) \) has a zero at 0 of order

(a) two, (b) one, (c) neither.

(d) Circle the correct choice.

Let \( f(x) = 1 - \cos(x^2) \). Then \( f(x) \) has a zero at 0 of order

(a) two, (b) four, (c) neither.

(e) Circle the correct choice.

Let \( f(x) = (x - 0.3)^2 \) so \( f \) has a zero of order 2 at \( x = 0.3 \). If the bisection method is started on the interval \([0, 1]\). Then the bisection method will

(a) converge with linear order, (b) converge with quadratic order, (c) fail.
(f) Circle the correct choice. 
Let \( f(x) = (x - 0.3)^2 \) so \( f \) has an zero of order 2 at \( x = 0.3 \). If Newton’s method is started with the initial value \( x_0 = 0.35 \). Then we expect that Newton’s method 
(a) converge with linear order, (b) converge with quadratic order, (c) fail.

(g) Circle the correct choice. 
Let \( f(x) = \cos x \) so \( f \) has an zero at \( x = \pi/2 \). If Newton’s method is started with the initial value \( x_0 = 1.5 \). Then we expect that Newton’s method 
(a) converge with quadratic order, (b) converge better than quadratic order, (d) converge worse than quadratic order.

(h) Circle the correct choice. 
The fixed point of a function \( y = f(x) \) has the following geometric interpretation. 
(a) The fixed points of \( f \) are the points where \( f \) crosses the \( x \)-axis. (b) The fixed points of \( f \) are the points where \( f \) crosses the \( y \)-axis. (c) The fixed points of \( f \) are the points where \( f \) crosses the line \( y = x \)-axis. (d) None of the above.