Homework One

DUE: Thursday, October 7, 1999, 8 pm.

1. For each of the following regular expressions, give as simple a regular expression as you can for the COMPLEMENT over \{0, 1\} of the language it denotes. None of the answers is very complicated. Hint: Describe the original language first in English.
   (a) 1*,
   (b) (1 \cup 10)*,
   (c) (0 \cup 1)*(01 \cup 10)(0 \cup 1)*.

2. Let $\Sigma = \{a, b\}$. Consider the language $L$ over $\Sigma$ given by the following rules:
   (1) $e \in L$,
   (2) if $x \in L$, then so are $axb, axa, bxa$, and $bxb$.
   (3) nothing else is in $L$.
   Show that any string of even length lies in $L$ by induction on the length of the string.

3. Show that $(A \cup B)^* = (A^* \cup B^*)^*$, where $A$ and $B$ are languages over the alphabet $\Sigma$ by showing that first $(A \cup B)^* \subseteq (A^* \cap B^*)^*$ then second $(A \cup B)^* \supseteq (A^* \cap B^*)^*$.

4. Let $\Sigma = \{a, b, c\}$. Which pairs of the following regular expressions are equal? Give an argument for your answer if they are equal by using the identities given in lecture. If the expressions are not equivalent, give an explicit string that lies in one language but not in the other.
   (a) $(a \cup b)^*a^*$ and $((a \cup b)a)^*$.
   (b) $(\emptyset^*)^*$ and $\epsilon$.
   (c) $((a \cup b) \cdot c)^*$ and $(ac \cup bc)^*$.
   (d) $b \cdot (ab \cup ac)$ and $(ba \cup ba) \cdot (b \cup c)$.

(H) — For Graduate Students — A regular expression is in disjunctive normal form if it of the form

$$(a_1 \cup a_2 \cup \ldots \cup a_n),$$

for some $n \geq 1$, where none of the $a$'s contains an occurrence of $\cup$. Give a proof using induction that every regular language is represented by a regular expression in disjunctive normal form.