As $x$ "gets huge", $\frac{x^2 + x}{2x^2 + 1}$

What happens?

In finite limits:
Translation -

As x gets larger and larger, what does \( \frac{2x^2 + 1}{x^2 + x} \) approach?

Algebra technique (avoid memorization)
As \( x \) gets large, what happens next

\[
\frac{x^2}{x^2 + x} \xlongequal{\text{divide top \& bottom}} \frac{1}{1 + \frac{1}{x}} = \frac{x}{x^2 + 1}
\]

By higher algebra.
\( p_n, q_n \) polynomials.

**Expression**

\[
\frac{p_n(x)}{q_n(x)}
\]

**Valid for any rational**

\[
\lim_{x \to \infty} \frac{2x^2 + 1}{x + 2} = 2
\]
\[ \frac{2x^2}{x^2 + 1} = \frac{\frac{2x^2}{x^2}}{1 + \frac{1}{x^2}} = \frac{2}{1 + \frac{1}{x^2}} = 2 \]

But work
Approximately \( \sqrt{2} \) \( \frac{t}{2} \) so approximately

\[
3 \approx \frac{(x \cdot \frac{t}{2} + t) \cdot x}{\left( \frac{x}{2} + \frac{t}{2} \right) \cdot x + \left( \frac{x}{2} + \frac{t}{2} \right)}
\]

\[
\lim_{x \to \infty} \frac{5x + 5}{2x + 1}
\]
\[ \lim_{x \to \infty} \left( x - \sqrt{4 + x^2} \right) \]
\[ \lim_{x \to \infty} x = \infty \]
Will take work/time to sort out philosophically.

Some limits are clear, visually, but...
\[
\lim_{x \to \infty} x = \infty \quad (2)
\]
\[
\lim_{x \to \infty} x^\alpha = \infty \quad (1)
\]

Fact: If \( a \) is any \( a > 1 \), and
VARIOUS LIMIT PROBLEMS

Let's use these facts for

\[
\lim_{x \to \infty} a^x = \infty \quad \text{and} \quad \lim_{x \to \infty} \log_a x = 0
\]
\[ \lim_{x \to \infty} \frac{x^2 + 1}{x^2 - 1} = \infty \]
\[ \lim_{x \to -\infty} \frac{e^x}{e^{-x} + x} \]
\[
\frac{\ln (3x) \ln 1 + 1}{\ln (x/1) \ln 1} \quad \alpha \in \mathbb{R}
\]
\[
\lim_{x \to \infty} \ln \left( \frac{1}{1 + 2x^2} \right) - \frac{1}{3x + 1}
\]