Example:

\[ f(x) = \cos(x) \text{ continuous everywhere} \]

Not defined:

\[ \text{All continuous, except where they are not defined} \]

Fact:
The standard 11 trig functions are invertible functions.
Just have to be careful with domains.

If \( f(x) \) is continuous, so is \( f^{-1}(x) \).
\[ \frac{2}{\pi} \sin x \]
Squeeze Theorem (or Tauberian Theorem)

Make complicated limits simple.
Suppose \( b(x), m(x), s(x) \) are three functions defined over \( C \) (but not necessarily on \( C \)).

\[
\lim_{x \to c} b(x) = L
\]

\[
\lim_{x \to c} m(x) = L
\]

\[
\lim_{x \to c} s(x) = L
\]

Therefore, for all \( i \) in \( C \), if

1. \( b(x) \geq m(x) \geq s(x) \)
2. \( \lim_{x \to c} s(x) = L \)
3. \( \lim_{x \to c} m(x) = L \)
4. \( \lim_{x \to c} b(x) = L \)

then \( \lim_{x \to c} L = L \) for all \( i \) in \( C \).
\[
\begin{align*}
0 \neq x & \quad \frac{x}{(x)} \sin x \\
\lim_{x \to 0} \frac{x}{(x) \sin x} & = \frac{0}{0} \\
\end{align*}
\]
(2)

(1) Ονομ

Do Better than Just Guess from a
Heeds a situation where you can
\[
\text{Area } \triangle ABC = \frac{1}{2} \cdot AB \cdot C
\]

\[
\text{Area } \triangle ABD = \frac{1}{2} \cdot AD \cdot BD
\]

\[
\text{Area } \triangle ABE = \frac{1}{2} \cdot AE \cdot BE
\]

\[
\text{Area } \triangle AED = \frac{1}{2} \cdot AD \cdot DE
\]

\[
\text{Area } \triangle ABC = \frac{2}{\pi R^2}
\]

\[
\text{Area } = \frac{2}{\pi R^2}
\]
\[
\lim_{\theta \to 0} \frac{\sin \theta}{\cos \theta} = \lim_{\theta \to 0} \frac{\theta - \sin \theta}{\theta - \cos \theta} \Rightarrow \frac{\theta - \sin \theta}{\theta - \cos \theta} = \frac{\theta - \sin \theta}{\theta - \cos \theta}
\]
The question: Are limits? Can use for?

Not obvious?

Limit

Suppose important

Measure?

\[
\lim_{x \to 0} \frac{x}{\sin(x)} = 1
\]
EXAMPLES: $\frac{\sin(6x)}{\sin(8x)}$
\[
\frac{\frac{3x^2}{\sin^2 x}}{26}
\]
Ex: \[
\frac{\tan(3x)}{\sin(2x)}
\]
\[
\begin{align*}
\text{Start} & \quad \frac{x}{1 - \cos(x)} = \frac{1 - \cos(x)}{x} = \frac{1 - \cos(2\theta)}{2x} \\
\text{(1)} & \quad \cos(2\theta) = 2\cos^2(\theta) - 1 = \cos^2(\theta) - \sin^2(\theta) = |1 - 2\sin^2(\theta)| \\
\text{(2)} & \quad \text{Use Double Angle Formulas (or Half Angle)} \\
\text{Another Limit:} & \quad \lim_{x \to 0} \frac{1 - \cos(x)}{x^2} = \frac{1}{2}
\end{align*}
\]
\[
\text{\textbf{Formula}}
\]
\[
\left( \frac{2}{x} \right) \frac{\sin x}{x} = \left( \frac{\sin x}{x} \right)^2 \frac{2}{x} = \frac{1}{2} \lim_{x \to 0} \frac{2}{x^2} \left( \frac{\sin x}{x} \right)^2
\]

\[
= \frac{1}{2} \lim_{x \to 0} \frac{2}{x^2} \left( \frac{\sin x}{x} \right)^2
\]
$$\lim_{h \to 0} \frac{1 - \cos(h)}{\sin(3h)}$$