1. General functions are complicated.
2. Linear functions are simple.
3. Linear approximations are integrals.
The point $A$ is located on the graph $y = f(x)$ at $x = a$.
1. \( L_a(x) \equiv f(x) x^2 a \)

2. DON'T CONFUSE \( a + x \)!

\[
L_a(x) = \frac{1}{1 + f'(a)(x-a)}
\]
\[
\begin{align*}
\text{let } 4.1 & \approx 4 \\
\text{so } 4.1 & \approx 4 \\
\frac{4}{4} & = \frac{t}{1} = f(a)\left(\frac{a}{4}\right) = 2 \\
\frac{2 + \sqrt{4}}{1} & = y = f(x) = \frac{x}{2} - 1 \\
\frac{x}{2} & = 4 - x \\
2x & = 8 \\
x & = 4 \\
f(4) = \frac{4}{4} & = 1
\end{align*}
\]
This sort of calculation?

Do you want to

When can you do as you want to
Since the error is small, we have

\[ e_{10} = 2.47560 \]

\[ 2 = 2.47548 + 0.241828 = 2.47548 \]

\[ e + (0.1) \cdot e = e + e (10 - 1.0) = e + e (9) \]

Let \( x(1) = 1.0 \) and \( \forall \in \mathbb{R} \), then

\[ f(x), \quad x \in \mathbb{R} \]

Ex: \( e \cup 1.0 \cdot e = \exists \]
as \( x \to a \).

\[
\frac{x-a}{M(x-a)} \sim \frac{\log(x)}{\log(x) - f(x)}
\]

In particular, \( \log(x) \) is much smaller.

If \( x-a \) small, \((x-a)^2\) is much smaller.

\[
M'(x) = M(x-a)^2 \frac{\log(x)}{\log(x) - f(x)}
\]

In fact,

\[
L_a(x) \sim f(x) \quad \text{if} \quad x \to 0
\]

How small is small?
\[ f(\pi) = -1.04 \]

\[ f(\pi) = 1.598 \]

\[ \frac{2x}{1 + \sin(x)} = \int f(x) \]

\[ s.t. \quad a = 0 \]

\[ \int x \cos(x) + \int = f(x) \]

Ex.
Differentials

\[ f(x) \approx f(a) + f'(a) (x-a) \]

\[ x = a + \Delta x \]

\[ f(a + \Delta x) \approx f(a) + f'(a) (a + \Delta x - a) \]

\[ f(a + \Delta x) - f(a) \approx f'(a) \Delta x \]

Or
\[ \Delta f \quad "\text{hard to compute}" \]
\[ df \quad "\text{easy to compute}" \quad (\text{if you know } f'(x)!) \]
\[ \Delta f \approx df \quad \text{if } \Delta x \text{ is small} \]
\[ dx = \Delta x \quad \text{then} \]
\[ \text{defined} \quad \text{(for us)} \]

This is a definition!

Its usefulness comes from \[ \Delta f \approx df. \]
Approximation.

Very Good

\[
\lim_{x \to 0} \frac{\sin x}{x} = 1
\]

Calculation are quite simple.

Linear Approximations and Differentiation

Various Important Examples
\[
\sqrt{\frac{4}{X-4}} \approx 2 + \frac{1}{4} \left( 4 + 8X - 4 \right) \sim \frac{2X}{4 + 8X} \sim \frac{X}{4 + 8X} \sim \frac{X}{4} \approx \frac{1}{4 + 8X} \sim 4 + 8X \sim 4 \quad (\text{if } X \sim 4)
\]

\[
\frac{f'(x)}{f(x)} \approx \frac{1}{4} = \frac{2}{8} = \frac{1}{4} = \frac{1}{4}
\]

\[
f'(a) = f(a) = 4 = 2
\]

Use linear approximation.

\[
f'(a) \approx \frac{x - a}{f(x) - f(a)} \approx \frac{1}{4 + 8X} \approx \frac{X}{4} \approx 4 + 8X \sim 4 + 8X \sim 4
\]
\[ X \circ \frac{\bar{A}}{1} = \]
\[ X p \frac{\bar{b}}{1} = \]
\[ X p \frac{\bar{B}}{1} = \]
\[ X p \frac{\bar{C}}{1} = \]
\[ X p (X, f) = \int p \quad \sim \quad 4 + 2 \bar{X} - 1 \bar{4} \]
\[ x \cdot \frac{s^2}{1} - \frac{s}{1} = \frac{(x \sigma + s)}{1} \]

\[ x \cdot \frac{s^2}{1} - \frac{s}{1} = \frac{s}{1} - \frac{(x \sigma + s)}{1} \]

\[ \frac{x^2}{1} = (x/\sigma) \quad s = x \quad \frac{x}{1} = (x/\sigma) \]

\[ x p (x/\sigma, \sigma) = x \sigma (x/\sigma) \sim (x/\sigma) - (x \sigma + x) \sigma \]

\[ s = x \quad \frac{x \sigma + s}{1} = (x/\sigma) \]
\[ \frac{1}{g(x+b)} \]