Show all your work on the exam paper, legibly and in detail, to receive full credit. The use of a calculator or any other electronic device is prohibited. **You may only use techniques used in sections 3.1-3.5.**

1. Find $\frac{dy}{dx}$
   a) $y = \pi^3$
   
   $y$ is a **constant**, so
   
   \[
   \frac{dy}{dx} = 0
   \]

   b) $y = -3x^{-8} + 2\sqrt{x}$
   
   \[
   \frac{dy}{dx} = \frac{d}{dx} \left( -3x^{-8} + 2\sqrt{x} \right) = -3 \cdot (-8)x^{-9} + 2 \cdot \frac{1}{2}x^{-1/2}
   \]
   
   \[
   = 24x^{-9} + x^{-1/2}
   \]

2. Find $\frac{d^2 y}{dx^2}$ if $y = 7x^3 - 5x^2 + x$.

   \[
   \frac{dy}{dx} = 21x^2 - 10x + 1
   \]
   
   \[
   \frac{d^2 y}{dx^2} = 42x - 10
   \]

   1 of 6
3. Using the definition of the derivative, find the derivative of \( f(x) = 3x^2 - 2x \).

\[
\frac{f(x+h) - f(x)}{h} = \frac{(3(x+h)^2 - 2(x+h)) - (3x^2 - 2x)}{h}
\]

\[
= \frac{3(x^2 + 2xh + h^2) - 2x - 2h}{h} - \frac{3x^2 - 2x}{h}
\]

\[
= \frac{3x^2 + 6xh + 3h^2 - 2x - 2h}{h} - \frac{3x^2}{h} + \frac{2x}{h}
\]

\[
= 6x + 3h - 2
\]

\[
f'(x) = \lim_{h \to 0} \frac{6x + 3h - 2}{h} = \left[ 6x - 2 \right]
\]

4. Suppose \( f(x) \) is a function with \( f(2) = -2 \) and \( \frac{df}{dx}(2) = -1 \). Suppose \( g(x) \) is defined by \( g(x) = x^2 + x^3 f(x) \). What is \( \frac{dg}{dx}(2) \)?

\[
\frac{dg}{dx} = 2x + 3x^2 f'(x) + x^3 \frac{df}{dx}
\]

\[
\frac{dg}{dx}(2) = 2 \cdot 2 + 3 \cdot 2^2 \cdot (-2) + 2^3 (-1)
\]

\[
= 4 - 24 - 8 = -28
\]
5 a) Find the derivative of the functions. Note: you do not have to simplify your answers.

\[ f(x) = \sec(x)\tan(x) \]

\[ f' = \left( \sec(x) \right)' \tan(x) + \left( \sec(x) \right) (\tan(x))' \]

\[ = \sec(x) \tan(x) \cdot \tan(x) + \sec(x) \sec^2(x) \]

\[ = \sec(x) \tan^2(x) + (\sec(x))^3 \]

5 b) \[ f(x) = \frac{5 - \cos(x)}{5 + \sin(x)} \]

\[ f' = \left( \frac{5 - \cos(x)}{5 + \sin(x)} \right)' \left( \frac{5 - \cos(x)}{5 + \sin(x)} \right) - \left( \frac{5 - \cos(x)}{5 + \sin(x)} \right) \left( \frac{5 + \sin(x)}{5 + \sin(x)} \right) \]

\[ = \frac{\sin(x) \left( 5 + \sin(x) \right) - \left( 5 - \cos(x) \right) \cos(x)}{\left( 5 + \sin(x) \right)^2} \]

\[ = 5\sin(x) + \sin^2x - 5\cos x + \cos^2x \]

\[ = \frac{\left( 5\sin(x) - 5\cos(x) + 1 \right)}{\left( 5 + \sin(x) \right)^2} \]
6 Given the function \( f(x) = \frac{1}{x} \):

6 a) Find the average rate of change of \( f(x) \) with respect to \( x \) over the interval \([1, 3]\).

\[
A.R.C. = \frac{f(3) - f(1)}{3 - 1}
\]

\[
= \frac{1}{3} - \frac{1}{1}
\]

\[
= \frac{1}{3}
\]

6 b) Find the instantaneous rate of change of \( f \) with respect to \( x \) at \( x = 2 \).

\[
f(x) = \frac{1}{x} = x^{-1}
\]

\[
f'(x) = -1 \cdot x^{-2} = \frac{-1}{x^2}
\]

\[
f'(2) = -\frac{1}{4}
\]
(prob 6, continued) 6 c) Find the equation to the line tangent to $f(x)$ at $x = 2$.

$$y = f(a) + f'(a)(x-a)$$

**Equation of Tangent Line Through $(a, f(a))$**

For us: $f(x) = \frac{1}{x}$, $f(2) = \frac{1}{2}$

$$f'(x) = -\frac{1}{x^2}$$

$$f'(2) = -\frac{1}{4}$$

$$y = \frac{1}{2} + -\frac{1}{4} (x - 2)$$

$$= \frac{1}{2} - \frac{x}{4} + \frac{1}{2} = 1 - \frac{x}{4}$$

**Equation of Tangent Line**

$$y = 1 - \frac{x}{4}$$
7 a) Find \( \frac{d^2 y}{dx^2} \) if \( y = x \cos x \)

\[
y' = 1 \cdot \cos x - x \sin x
\]

\[
y'' = \cos x - x \sin(x)
\]

\[
y''' = -\sin(x) - \left[ \sin(x) + x \cos(x) \right]
\]

\[
= -2\sin(x) - x \cos(x)
\]

(bonus) 7 b) (harder!!) : If \( y = x \cos x \), what is \( \frac{d^4 y}{dx^4} \) ?

**IS A PATTERN:**

\[
\frac{d^3 y}{dx^3} = -2 \cos(x) - \left[ \cos(x) + x (-\sin(x)) \right]
\]

\[
= -3 \cos(x) + x \sin(x)
\]

\[
\frac{d^4 y}{dx^4} = 3 \sin(x) + \left[ \sin(x) + x \cos(x) \right]
\]

\[
= 4 \sin(x) + x \cos(x)
\]

\[
\frac{d^5 y}{dx^5} = 4 \cos(x) + \left[ \cos(x) - x \sin(x) \right]
\]

\[
= 5 \cos(x) - x \sin(x)
\]

Now \( \frac{d^4 y}{dx^4} = \cos(x) - x \sin(x) \), so it looks like it "repeats" after 4 derivatives, except for coefficient.

**GUESS:** \( \frac{d^4 y}{dx^4} = 41 \cos(x) - x \sin(x) \)