Show all your work on the exam paper, legibly and in detail, to receive full credit. The use of a calculator or any other electronic device is prohibited. You may only use techniques discussed in text up to the end of section 2.2.

(5 points each) Find the limits.

\[ \lim_{x \to -\infty} \frac{x - 2}{x^2 + 2x + 1} = \frac{x - 2}{x^2 + 2x + 1} = \frac{1}{x^2} (x - 2) \]

\[ \lim_{x \to -\infty} \frac{\sqrt{5x^2 - 2}}{x + 3} = \frac{\sqrt{5x^2 - 2}}{x + 3} = \frac{\sqrt{5 - \frac{2}{x^2}}}{1 + \frac{3}{x}} \]

\[ \lim_{x \to +\infty} \frac{\sqrt{5x^2 - 2}}{x + 3} = \frac{\sqrt{5 - \frac{2}{x^2}}}{1 + \frac{3}{x}} \]

\[ \lim_{x \to -\infty} f(x) = \frac{\sqrt{5 - \frac{2}{x^2}}}{1 + \frac{3}{x}} = -\sqrt{5} \]

\[ \lim_{x \to +\infty} f(x) = \frac{\sqrt{5 - \frac{2}{x^2}}}{1 + \frac{3}{x}} = \sqrt{5} \]
(5 points each) Find the limits

\[
\lim_{x \to \infty} \ln \left( \frac{1}{x^2} \right) = \ln \left( \frac{1}{x^2} \right) = - \ln (x^2)
\]

\[
= -2 \ln (x)
\]

\[
\lim_{x \to \infty} -2 \ln (x) = \text{"-\infty" OR DNE (DOES NOT EXIST)}
\]

\[
\lim_{x \to 0^+} e^{\sin x}
\]

\[
f(x) = e^{\sin x} \quad \text{"Plug in" EVALUATION WORKS: } e^0 = 1
\]

\[
\lim_{x \to 0} \frac{x^2}{1 - \cos x}
\]

\[
(1) \text{ IF YOU MISMOLED } \frac{1 - \cos x}{x^2} \Rightarrow \frac{1}{2}
\]

THEN \[
\lim_{x \to 0} \frac{x^2}{1 - \cos x} = \frac{1}{2} = 2
\]

OR \[
f(x) = \frac{x^2}{1 - \cos x} \cdot \frac{1 + \cos x}{1 + \cos x} = \frac{x^2 (1 + \cos x)}{1 - \cos^2 x} = \frac{x^2 (1 + \cos x)}{\sin^2 x}
\]

\[
\lim_{x \to 0} \sin \left( \frac{1}{x} \right) = \frac{x}{\sin x} \cdot \frac{x}{\sin x} (1 + \cos x), \quad \lim_{x \to 0} f(x) = 1 \cdot 1 \cdot 2
\]

\[
\lim_{x \to 0^+} \sin \left( \frac{1}{x} \right) \text{ DOES NOT EXIST. IT JUST OSCILLATES BETWEEN -1 AND 1}
\]
(5 points each) Find values of $x$, if any, at which $f$ is not continuous. If there are no such points, explain why.

$$f(x) = \frac{x^2 + 6x + 9}{|x| + 3} \quad \text{f(x) is always continuous. It is the ratio of two continuous functions, and the denominator never vanishes.}$$

$$f(x) = \frac{x + 2}{x^2 - 4} \quad \text{f(x) is the ratio of two continuous functions. The only x values which make the denominator equal to zero are } x^2 - 4 = 0 \quad \text{points at which f is not cont.}$$

$$x^2 = 4 \quad \left\{ \begin{array}{l} x = \pm 2 \end{array} \right.$$
(5 points each) Given the function $f(x) = x^2 - 1$.

Find the average rate of change of $f$ with respect to $x$ over the interval $[-1, 2]$.

$$\frac{f(2) - f(-1)}{2 - (-1)} = \frac{(4) - (1) - (1 - 1)}{3} = 1$$

Find the instantaneous rate of change of $f$ with respect to $x$ at the point when $x = -1$.

$$\lim_{x \to -1} \frac{f(x) - f(-1)}{x - (-1)} = \frac{(x^2 - 1) - (0)}{x + 1} = \frac{(x+1)(x-1)}{x+1} = x - 1$$

Find the equation of the tangent line to $f$ at the point where $x = -1$.

$f(-1) = 0$: SLOPE OF TANGENT LINE IS $-2$; so $y = y_0 + m(x-x_0)$

OR $y = 0 + (-2)(x - (-1))$

$= -2x - 2 = -2(x + 1)$
(10 points) Find the value(s) for the constant $k$ that makes
\[ f(x) = \begin{cases} \frac{\sin(kx)}{x}, & x < 0 \\ 3x + 2k^2, & x \geq 0 \end{cases} \]
continuous at $x = 0$.

\[
\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} 3x + 2k^2 = 2k^2
\]

\[
\lim_{x \to 0^-} \frac{\sin(kx)}{x} = \lim_{x \to 0^-} \frac{\sin(kx)}{kx} \cdot k = k \cdot 1 = k
\]

so we need $2k^2 = k$ or $k = 0, k = \frac{1}{2}$

(10 points) Using the fundamental definition of the derivative, find the derivative of $f(x) = x^2 - x$.

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
\]

look at: \[
\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - x^2 - x}{h}
\]

= \[
\frac{(x^2 + 2xh + h^2) - x - h - x^2 + x}{h}
\]

= \[
\frac{2xh + h^2 - h}{h} = 2x + h - 1
\]

so \[ f'(x) = \lim_{h \to 0} 2x + h - 1 = 2x - 1 \]
For each "lettered" function graph on the left hand side, find the "numbered" graph on the right hand side which corresponds to the graph of its derivative. Place your answer in the space provided. (2 points each)

a) [Graph A]
   - [Graph 1]

b) [Graph B]
   - [Graph 2]

c) [Graph C]
   - [Graph 3]

d) [Graph D]
   - [Graph 4]

e) [Graph E]
   - [Graph 5]

f) [Graph F]
   - [Graph 6]
For each "lettered" function graph on the left hand side, find the "numbered" graph on the right hand side which corresponds to the graph of its derivative. Place your answer in the space provided. (2 points each)

a)_____

b)_____

c)_____

d)_____

e)_____

f)_____

1

2

3

4

5

6

6 of 6