1. Consider the functions \( f(x) = 4 - x^2 \) and \( g(x) = \sqrt{6 - x} \)
   a. **[5 Points]** Find the domain of \( f \) and the domain of \( g \).

   b. **[5 Points]** Find the domain of \( \frac{g}{f} \).

   c. **[5 Points]** Find the domain of \( f \circ g \).

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Show all your work on the exam paper, legibly and in detail, to receive full credit. The use of a calculator or any other electronic device is prohibited. You may only use techniques discussed to date in class.
1. (continued) Once again, consider the functions \( f(x) = 4 - x^2 \) and \( g(x) = \sqrt{6 - x} \).

d. **[6 Points]** Use the fundamental definition of the derivative to find \( f'(x) \).
   (You may NOT use l’hopital’s rule for this problem!)

e. **[5 Points]** Find the instantaneous rate of change of \( g \) with respect to \( x \) at \( x = 2 \).

f. **[5 Points]** Find the average rate of change of \( g \) with respect to \( x \) over the interval \([2, 5]\).
2. [8 Points Each] Evaluate the limits:

a. \( \lim_{x \to 2} \frac{x^2 - 5x + 6}{x^2 + x - 6} \)

b. \( \lim_{x \to 0} \frac{\cos x}{x} \)

c. \( \lim_{x \to -\infty} \left( x \sin \frac{3}{x} \right) \)

d. \( \lim_{x \to +\infty} \left( 1 - \frac{5}{x} \right)^x \)
3. [6 Points] On the axes provided below, sketch the graph of a function $f$ that has all of the following characteristics:

- $f(2) = 3$
- $f$ is continuous everywhere except at $x = 2$
- $f$ is not differentiable at $x = -1$
4. [8 Points Each] Find $\frac{dy}{dx}$ (You do NOT need to algebraically simplify your answers.)

   a. $y = 2x^5 - \frac{4}{x^2} + \sqrt{x} - \ln 3$

   b. $y = \frac{2x^2 + 7}{3x^3 - \cos x}$

   c. $y = \cot^2 x^3$
5. [8 Points Each] Find \( \frac{dy}{dx} \) (You do NOT need to algebraically simplify your answers.)

a. \( y = \tan^{-1}(x^3) \)

b. \( y = \sqrt{x} e^{\sqrt{x}} \)
6. [8 Points Each] Find \( \frac{dy}{dx} \) (You do NOT need to algebraically simplify your answers.)

a. \( x \sin y = y^3 \)

b. \( y = x^{\sec x} \)
7. **[7 Points]** Find the equation of the tangent line to the graph of \( y = f(x) \) at \( x = -3 \) if 
\[
f(-3) = 2 \quad \text{and} \quad f'(3) = 5.\]

8. **[8 Points]** Use an appropriate local linear approximation to estimate the value of \( \sqrt{24} \).
9. **[10 Points]** The derivative of a function is given to you. Find all critical points of $f$ and at each critical point use the first derivative test to determine whether a relative maximum, relative minimum, or neither occurs. Assume that $f$ is continuous everywhere.

$$f'(x) = (x - 1)^2 (x - 3)^3 (x - 5)$$

10. **[10 Points]** Use the second derivative test to determine the locations of all relative extrema of $f(x) = 2\cos x + x$ on the interval $[0, \pi]$. 
11. [16 Points] On the axes provided on the next page, sketch the graph of the given function \( f \) and identify the locations of all critical points and inflection points. Label any intercepts and asymptotes, if any. The first and second derivatives are given to you.

\[
\begin{align*}
f(x) &= \frac{1}{3} (4 - x) \\
f'(x) &= \frac{4(1 - x)}{3x^3} \\
f''(x) &= \frac{-4(x + 2)}{9x^3}
\end{align*}
\]

Hint: The point \((-2, -7.5)\) is on the graph of \( f \).
11. On the axes provided below, sketch the graph of the given function $f$ and identify the locations of all critical points and inflection points. Label any intercepts and asymptotes, if any. The first and second derivatives are given to you.

$$f(x) = x^3(4 - x)$$
$$f'(x) = \frac{4(1 - x)}{3x^3}$$
$$f''(x) = \frac{-4(x + 2)}{9x^3}$$

Hint: The point $(-2, -7.5)$ is on the graph of $f$.

NOTE: All necessary information will fit on these axes! Label appropriately.
12. [12 Points] Find the absolute maximum and minimum values of \( f(x) = \ln x - x \), if any, on the interval \([\frac{1}{4}, 4]\), and state where those values occur. If there is no absolute maximum or minimum value, say so. **Hint:** \( \ln 4 = 1.4 \)

13. [12 Points] An open box is to be made from a 16-inch by 30-inch piece of cardboard by cutting out squares of equal size from each of the four corners and bending up the sides. What size should the squares be in order to obtain the box with the largest volume?