Exam Two  
MATH 122  
Winter, 2009

Name _______________________________    Section _____

Show all your work on the exam paper, legibly and in detail, to receive full credit. The use of a calculator or any other electronic device is prohibited. You may only use techniques discussed to date in class.  You must simplify all answers unless you are explicitly instructed not to.

1. (10 pts each) Evaluate the integrals:

   a. \[ \int x^3 \ln x \, dx \]

   b. \[ \int x^2 \sin x \, dx \]
2a) (5 pts) Draw the region enclosed by the curves $y = x$, $y = 4x$, and $y = 2$. Be sure to give the coordinates of the intersection points of the curves.

2b) (5 pts) Write down the integral required to compute the area of the region enclosed by the curves of part a), as an integral with respect to "x". You will need to split it into two pieces. Do not evaluate the integrals.

2c) (5 pts) Write down the integral required to compute the area of the region enclosed by the curves of part a), as an integral with respect to "y".

2d) (5 pts) Evaluate the integral in part c)
3 (10 pts) Find the area of the region in the first quadrant enclosed by the curves \( x = 0 \), \( y = 2 \) and \( y = \frac{1}{\sqrt{1-x^2}} \). Treat it as an integral with respect to \( x \).

**Hint:** The two curves intersect at the point \( x = \frac{\sqrt{3}}{2}, y = 2 \).
Note: For the problems on this page you do NOT have to evaluate the integrals and you do NOT have to algebraically simplify the integrands.

4a. (10 pts) Find the definite integral that represents the volume of the solid that results when the volume of the solid enclosed by the curves \( y = \sqrt{x} \), \( y = 0 \), and \( x = 9 \) is revolved about the \( x\)-axis.

4b. (5 pts) Find the definite integral that represents the volume of the solid that results when the volume of the solid enclosed by the curves \( y = \sqrt{x} \), \( y = 0 \), and \( x = 9 \) is revolved about the \( y\)-axis.
5a) (10 pts) A spring whose natural length is 15 cm exerts a force of 45 N when stretched to a length of 20 cm. Find the integral that represents the work done in stretching the spring 3 cm beyond its natural length.

5b) (5 pts) Evaluate the integrals of part 2b). You should get the same answer as 2d)
6) (10 pts) A cone-shaped reservoir is 20 ft in diameter across the top and 15 ft deep. If the reservoir is filled to a depth of 10 ft, find the integral that represents the work done in pumping all the water to the top of the reservoir. Use 62.4 lb/ft³ as the weight density of water. Set up the relevant integral - DO NOT EVALUATE THE INTEGRAL.
Bonus Problem

7a) (2 pts) Write down an integral with respect to $y$ which gives the area in problem 3).

7b) (3 pts) Evaluate the integral in 6b). You should get the same answer as in problem 3).
Space for additional work