QUIZ 12 ANSWERS

1. A) Future value (compounding) = Present value $\times (1 + \frac{r}{m})^{mt}$
1. B) Present value (compounding) = Future value $\times (1 + \frac{r}{m})^{-mt}$
1. C) Accumulated amount (of an annuity) = Payment $\times \frac{(1 + \frac{r}{m})^{mt} - 1}{r/m}$
1. D) Payment = Accumulated amount (of an annuity) $\times \frac{r/m}{(1 + \frac{r}{m})^{mt} - 1}$
1. E) Effective rate (or APY) = $(1 + \frac{r}{m})^m - 1$
1. F) Simple interest = Deposit $\times$ Rate $\times$ Time
1. G) Present value of an annuity = Payment $\times \frac{1 - (1 + \frac{r}{m})^{-mt}}{r/m}$
1. H) Payment = Present value of an annuity $\times \frac{r/m}{1 - (1 + \frac{r}{m})^{-mt}}$
1. I) Debt (present value) = Payment $\times \frac{1 - (1 + \frac{r}{m})^{-mt}}{r/m}$
1. J) Payment = Debt $\times \frac{r/m}{1 - (1 + \frac{r}{m})^{-mt}}$
1. K) Term of an annuity = $\frac{\ln \left(1 + \frac{A/r}{m}\right)}{m \ln \left(1 + \frac{r}{m}\right)} = \frac{\log \left(1 + \frac{A/r}{m}\right)}{m \log \left(1 + \frac{r}{m}\right)}$
1. L) Number of periods of an annuity = $\frac{\ln \left(1 + \frac{A/r}{m}\right)}{\ln \left(1 + \frac{r}{m}\right)} = \frac{\log \left(1 + \frac{A/r}{m}\right)}{\log \left(1 + \frac{r}{m}\right)}$
1. M) Balance unpaid after $t_0$ years = Payment $\times \frac{1 - (1 + \frac{r}{m})^{-m(t-t_0)}}{r/m}$
1. N) Sinking fund payment = Accumulated amount $\times \frac{r/m}{(1 + \frac{r}{m})^{mt} - 1}$

2. A)

\[
\begin{align*}
-2x + 2y &= -20 \\
x - y &= 10
\end{align*}
\]

The system is dependent, it has infinitely many solutions given by $x = t, y = t - 10$, where $t$ runs from $-\infty$ to $+\infty$. Both equations describe the same line $y = x - 10$ (see the graph).

2. B)

\[
\begin{align*}
3x - 2y &= 30 \\
-6x + 4y &= 30
\end{align*} \iff \begin{align*}
3x - 2y &= 30 \\
3x - 2y &= -15
\end{align*} \iff \begin{align*}
y &= 1.5x - 15 \\
y &= 1.5x + 7.5
\end{align*}
\]
This system is inconsistent, it has no solutions. Equations describe two (distinct) parallel lines (see the graph).

2. C) Let \( n \) be the number of nickels and let \( d \) be the number of dimes. Then
\[
\begin{align*}
\begin{cases}
  n + d = 60 \\
  5n + 10d = 430
\end{cases} & \iff \\
\begin{cases}
  n + d = 60 \\
  n + 2d = 86
\end{cases} & \iff \\
\begin{cases}
  n + d = 60 \\
  d = 26
\end{cases} & \iff \\
\begin{cases}
  n = 34 \\
  d = 26
\end{cases}
\end{align*}
\]
The system has a solution and this solution is unique. The equations \( d = -n + 60 \) and \( d = -.5n + 43 \) describe two lines with a single point \((34, 26)\) in common (see the graph).