Math 121 Calculus 1 Fall 2007
Outcomes List

We have covered a lot of material this quarter. This outcomes list summarizes what skills and knowledge you should have acquired thru the course, and what sort of problems to expect on the final exam. The problems here are representative, although we do not guarantee that the problems on the exam will look exactly like the ones here. Most are from homework problems assigned during the quarter; and some are extra problems from the book.

Chapter 1: Pre-calculus fundamentals. In this chapter we quickly review the knowledge we expect students to have as we approach the rest of the course.

1.1 Define functions and determine when a relation among quantities constitutes a function, i.e., by the vertical line test. Given a function, determine its domain and range. (1.1: 19,21,31,32,33)

1.3 Define operations on functions, such as addition of functions, multiplication, and composition, and graph transformations such as translation, dilation, and reflection. Define and recognize even and odd functions; quickly recognize some common even and odd functions, such as powers of x. Use these symmetries to draw conclusions about the behavior of such a function. (1.3: 1,29,41,53,55)

1.5 Given a function, determine whether it has an inverse and, if so, what this inverse is. Apply all the objectives above to such inverses. Relate the graph of a function and its inverse. Note particularly the important inverse trigonometric functions; apply the triangle method to evaluate such functions. (1.5: 8,24,30,38)

1.6 Define the number e and give its approximate value. Describe what is meant by exponential growth or decay, and relate these to exponential functions \( b^x \) and \( b^{-x} \). Relate exponentiation to logarithms, especially the natural log. State and use the basic properties of exponents and logs that are useful for calculation. (1.6: 11,17,21,23,33,34)

Chapter 2: The Limit. The limit is the basic object defining calculus. In this chapter we define the limit and develop techniques for finding limits of various functions.

2.1 Calculate the limit of a function \( f \) of a variable \( x \), as \( x \) approaches some finite value \( c \). Determine when such a limit does not exist, as a one-sided or two-sided limit. In either case, describe the behavior of \( f \) in the vicinity of such a point. Include an understanding of what is meant by increase or decrease without bound in the vicinity of such a point, or a vertical asymptote. (2.1: 5,6)

2.2 Discuss how the limit interacts with basic arithmetic operations such as addition, multiplication, and division. Take limits of functions involving especially quotients or radicals; define some indeterminate forms. (2.2 : 7,11,31,37)
2.3 Determine whether a function \( f \) has a limit as \( x \) approaches positive or negative infinity, that is, its end behavior. If it does, give the value and relate to the existence of a horizontal asymptote. If not, determine whether the function increases or decreases without bound. Apply specifically to quotients of polynomials. (2.3: 26, 28, 50, 55)

2.5 Define continuity of a function \( f \), at a point and over an interval. Determine where a given function is continuous or discontinuous. State the Intermediate Value Theorem, and use it, for example, to find roots. (2.5: 27, 29, 41)

2.6 Describe the continuity properties of the trigonometric and inverse trigonometric functions. Use the Squeeze Theorem to evaluate limits of some such functions. Be familiar with frequently-used limit behaviors such as the ratios \( \{\sin x\}/x \) and \( \{1 - \cos x\}/x \). (2.6: 35, 41, 43, 47)

Chapter 3: The Derivative. The first tool of calculus is the derivative. In this chapter we define it, find it for various simple functions and combinations of these, and develop a toolkit for taking more complicated derivatives. Near the end, we apply these tools to some model situations.

3.1 Define the tangent line to a function \( f \) at a point on its graph. Describe it geometrically, in terms of its direction and shape, and numerically, in terms of its slope and value in the vicinity of that point. Relate these to the analogous quantities for \( f \). Give the slope of such a tangent line a physical meaning in terms of velocity of a particle or rate of change of a modeled quantity. (3.1: 13, 15)

3.2 State, with rigorous accuracy, the definition of the derivative to a function \( f \) at a point \( x = c \). Use this definition to find derivatives of particular functions at given points in the domain of \( f \). Similarly, use the definition to calculate derivatives as a function of values of \( x \) ranging over the entire domain. Relate the construction of this concept to the tangent line viewed as a limit, with particular regard to the slope of that tangent line. Determine whether a function is differentiable at a given point. Relate differentiability and continuity. (3.2: 15, 29, 44)

3.3 Give the derivative of \( x^n \) for any real \( n \). Relate differentiation to real linear combinations of functions: sums, differences, and constant multiples. Define and calculate higher-order derivatives. (3.3: 11, 13, 29, 43, 52)

3.4 State and use the product and quotient rules for differentiation. (3.4: 11, 23, 26, 29)

3.5 Give the derivative of any trigonometric function. Be able to derive these results using the limits discussed earlier, in combination with the limit rules and several trigonometric identities. (3.5: 13, 17, 29, 33)

3.6 Use the chain rule to differentiate compositions of functions. (3.6: 11, 29, 51, 62)
3.7 Solve related rates problems: given a system of interacting quantities and knowledge of how one is changing, relate the quantities and determine how another quantity in the system is changing. (3.7: 7,25,31,37)

3.8 Describe the local linear approximation of a function near a given point. Relate the value and behavior of this approximation to the value and behavior of the original function. Use local linear approximation, or differentials, to estimate values of f near a convenient known value, and to describe propagation of error in measurements, both absolute and relative. (3.8: 41,43)

**Chapter 4: Derivatives of Transcendental Functions.** In this chapter we calculate derivatives for some trickier functions, and use our wider array of tools to round out our array of techniques.

4.1 Use implicit differentiation to determine dy/dx when x and y are related implicitly, without explicitly solving for y. (4.1: 13, 24, 29)

4.2 Differentiate functions log_b x, in particular ln x. (4.2: 13, 15, 30, 37)

4.3 Given an invertible function f, calculate the derivative of f^-1 without explicitly obtaining f^-1. Use the ideas of these three sections to differentiate exponential functions and the inverse trigonometric functions. (4.3: 17,18,27,43,61)

4.4 Use the techniques of differentiation encountered thus far to employ L'Hopital's rule for the evaluation of indeterminate limits. Given a limit not necessarily of indeterminate form, convert it to such a form (if possible) and evaluate. (4.4: 13, 21, 33, 53)

**Chapter 5: Analysis of functions.** In this chapter we take everything we have learned so far about limits, continuity, rates of change, etc., and apply them to develop a mental toolkit for visualizing and conceptualizing the behavior of a function, translating equations into useful qualitative understanding of a system.

5.1 Use the first derivative to determine where a function is increasing, decreasing, or constant as x increases. Identify critical points and stationary points. Define the concavity of a function and use the second derivative to determine intervals of concavity up and down, and the location of inflection points. (5.1: 17,29,31)

5.2 Define relative extrema and locate them for a given function. Relate these to the behavior of polynomials at simple or multiple roots. Combine with an understanding of the end behavior of polynomials ("dominant term") to quickly sketch graphs of polynomials. (5.2: 17,39,41, 61)

5.3 Determine whether a function has cusps or vertical tangent lines, or asymptotes (vertical, horizontal, oblique, or curvilinear). Sketch rational functions using this and extremum information. (5.3: 29, 41, 47)
5.4 Locate the absolute maxima and minima of a function over a given interval. (5.4: 11, 17, 35, 39)

5.5 Use the above to solve optimization problems: given a system of related quantities, one of which is to be optimized in some sense, find the set of values for the quantities that optimizes the desired value. (5.5: 11, 29, 41, 51)